Impact of Multiple Channels and Radios on the Capacity, Channel Assignment, and Flow Allocation in Wireless Mesh Networks

A Project Report

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by

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under the guidance of

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Certificate

This is to certify that the project entitled **Impact of Multiple Channels and Radios on the Capacity, Channel Assignment, and Flow Allocation in Wireless Mesh Networks** submitted by **Vibhav Bukkapatanam** in partial fulfillment of the requirements for the award of the degree of **Bachelor of Technology**, is a bona-fide record of work carried out by him under my supervision and guidance at the Department of Computer Science and Engineering, Indian Institute of Technology Madras.

Place: Chennai [Prof. C. Siva Ram Murthy]
Date:

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Date:

Abstract

Wireless Mesh Networks (WMNs) have recently gained significant attention, since they address important problems such as: the problem of growing bandwidth demand in networks with scarcity of the available spectrum and the need for flexible, easily deployable, self configuring, and adaptable networks. WMNs mix the robustness of cellular networking with the flexibility of ad hoc networking. A very important consideration for the widespread deployment of such broadband WMNs is the support of bandwidth intensive applications such as streaming video and real time voice and video traffic. In order to support such traffic, the network must provide QoS guarantees on the connection. A few of the important aspects to consider while providing such guarantees is the channel assignment algorithm which effectively utilizes and distributes the network resources and the routing protocol which chooses paths that satisfy the QoS requirement. A problem of considerable interest is the estimation of theoretical guarantees that can be provided by any QoS scheme. Such estimates give us clarity on the maximum guarantees that can be provided and also act as a reference with which we can compare the performance of existing protocols.

In this work, we theoretically analyze the performance in terms of end-to-end call acceptance in Multi-Channel Multi-Radio (MC-MR) WMNs using queueing theory techniques. We study the impact of the routing protocol and the channel assignment algorithm on the end-to-end call acceptance and address the issue of providing deterministic QoS guarantees to a designated set of routers. We have adopted a TDMA based WMN as our base framework. We consider the case of differentiated class of users and derive upper (P_{Acc}^{Max}) and lower (P_{Acc}^{Min}) bounds for the probability of call acceptance (P_{Acc}) and theoretically estimate the probability of system saturation (P_{Sat}) (which is the probability that no more new calls can be accepted in the network). Through simulations, we study the dependence of the (P_{Acc}) on the number of radios in each router ($\mathcal{K}$), the number of channels available in the network (C), the network load (ρ), the routing protocol used, and the channel assignment algorithm. We also study the effect of Weighted Cumulative Expected Transmission Time (WCETT) routing metric and compare its performance with the Shortest Path (SP) routing protocol. The increase in P_{Acc} with the WCETT metric emphasizes the need to consider better routing metrics in an MC-MR WMN.

We then look into the issue of improving the system throughput of WMNs, when both non-overlapped channels and partially overlapped channels are used, by proposing a channel assignment and flow allocation algorithm. A major factor which is responsible for the high throughput of WMNs is the usage of multiple interfaces in each mesh router, along with the availability of multiple channels for transmission in the network. Recent studies have shown that the system throughput can further be increased, when both non-overlapped channels and partially overlapped channels are used. It has also been shown in the literature that there is a non-linear increase in system capacity, with the increase in the number of radios at each mesh router. However, due to the limited number of channels available, channel contention is possible, leading to collisions and interference. Thus, in order to effectively utilise the multiple radios and multiple channels, efficient channel assignment algorithms are needed. Since the traffic characteristics of a multihop WMN are quite different from a single hop wireless network, we consider maximizing aggregate useful end-to-end throughput and minimize queueing delay in the network, instead of the sum of link capacities. Through simulations, we show that our Mixed Integer Linear Program (MILP) formulation makes efficient use of the spectrum, by providing superior channel assignments and flow allocations, with the addition of partially overlapped channels. In order to scale our algorithm to bigger network topologies, we also propose a polynomially bounded heuristic algorithm.

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Abbreviations

BACnet	Building Automation and Control network
BER	Bit Error Rate
CDF	Cumulative Distribution Function
DSL	Digital Subscriber Line
DSSS	Direct-Sequence Spread Spectrum
ETT	Expected Transmission Time
IEEE	Institute of Electrical and Electronics Engineers
ILP	Integer Linear Program
IP	Internet Protocol
IR	Interference Range
LIM	Link Interference Matrix
LMST	Local Minimum Spanning Tree
MAC	Medium Access Control
MC-MR	Multi-Channel Multi-Radio
MILP	Mixed Integer Linear Program
MR-LQSR	Multi-Radio Link Quality Source Routing
OFDM	Orthogonal Frequency Division Multiplexing
QoS	Quality of Service
SINR	Signal to Interference plus Noise Ratio
SP	Shortest Path
TDMA	Time Division Multiple Access
TR	Transmission Range
UDP	User Datagram Protocol
WCETT	Weighted Cumulative Expected Transmission Time
WLAN	Wireless Local Area Network
WMN	Wireless Mesh Network

Notation

K	Number of classes of calls
C	Number of channels available in the network
$\mathcal{K}$	Number of radios in each node
ϵ	Routing protocol constant
ρ	Network load
P_{Acc}	Call acceptance probability
P_{Sat}	System saturation probability
$\mathcal{N}$	Set of mesh nodes
N	Number of mesh nodes
$\mathcal{E}$	Set of logical links in the network
E	Number of logical links in the network
A	Network area
$R(j)$	Interference range of a node j
B	Number of time slots in a super-frame
λ_k	Mean arrival rate of calls of class k
μ_k	Parameter of the exponentially distributed call duration of calls of class k
$\alpha_c(j)$	Fraction of edges in $R(j)$ that are assigned channel c
$\lambda_{k,c}(j)$	Mean arrival rate of class k calls in $R(j)$ in channel c
$n_{k,c}$	Number of class k calls being served by channel c in $R(j)$
$n_{k,c,V}$	Number of class k calls of type-V being served by channel c in $R(j)$
$\rho_{k,c}(j)$	Load due to class k calls being served by channel c in $R(j)$
$\rho_{k,c,V}(j)$	Load due to class k calls of Type-V being served by channel c in $R(j)$
$\mathcal{D}$	Deterministic guarantee limit
$\mathcal{T}$	Set of communications sessions in the network
t_{ij}	Throughput demand of a flow from node i to node j
x_{ij}	Achieved throughput a flow from node i to node j
$[o]_{mn}$	Channel overlap factor between channels m and n
G_{ab}	Channel gain of for nodes a and b
P_a	Transmission power of node a
r_{ab}	Euclidean distance between nodes a and b
N_0	Thermal noise power

$R^{e}_{ij,k}$	Indicator variable for routing
$\bar{\mathbf{w}}_{\mathbf{e}}$	Channel assignment vector for a link e
$\bar{\mathbf{y}}_{\mathbf{n}}$	Interface assignment vector for a node n
c_0	Nominal data transmission rate
$l(e)$	Aggregate load on a link e
$l_I(e)$	Aggregate load in the interference range of a link e
$W_I(e,c)$	Interference weight of a link e on channel c

CHAPTER 1

Introduction

1.1 Wireless Mesh Networks

Today's wireless networks face problems of growing bandwidth demand with scarcity of the available spectrum, along with the need to be flexible, easily deployable, self configuring, and adaptable. Wireless Mesh Networks (WMNs) [1] have recently gained significant attention, because they address all of the above problems. WMNs have several advantages such as low up-front cost, easy network maintenance, robustness, and reliable service coverage. WMNs mix the robustness of cellular networking with the flexibility of ad hoc networking. Emergence of such networks has also been spurred by the development of recent standards such as the IEEE 802.16 [2] and other special working groups such as IEEE 802.11s in the context of IEEE 802.11 networks [3].

1.1.1 Architecture of Wireless Mesh Networks

WMNs consist of two kinds of nodes: mesh routers and mesh clients. A mesh router supports all functions of a conventional wireless router, and in addition to that has specific additional functions to support mesh networking. To improve the flexibility and scalability of mesh networks, a mesh router also has multiple wireless interfaces built on either the same or different wireless access technologies. In contrast, the mesh clients typically have only one wireless interface, without any gateway or bridge functionality. As a consequence, the hardware platform and the software for mesh clients can be much simpler than those for mesh routers. Typical examples of mesh clients include, laptop/desktop PC, pocket PC, PDA, IP phone, RFID reader, and other devices.

WMN architectures can be classified into three broad categories, depending on the functionality of the mesh nodes:

- **Infrastructure/Backbone WMNs:** Infrastructure WMNs (Figure 1.1) are characterized by an ad hoc backhaul, which is the infrastructure part of the network to which the clients connect. The backhaul, mainly consists of wireless mesh routers that are interconnected in an ad hoc manner. The clients are

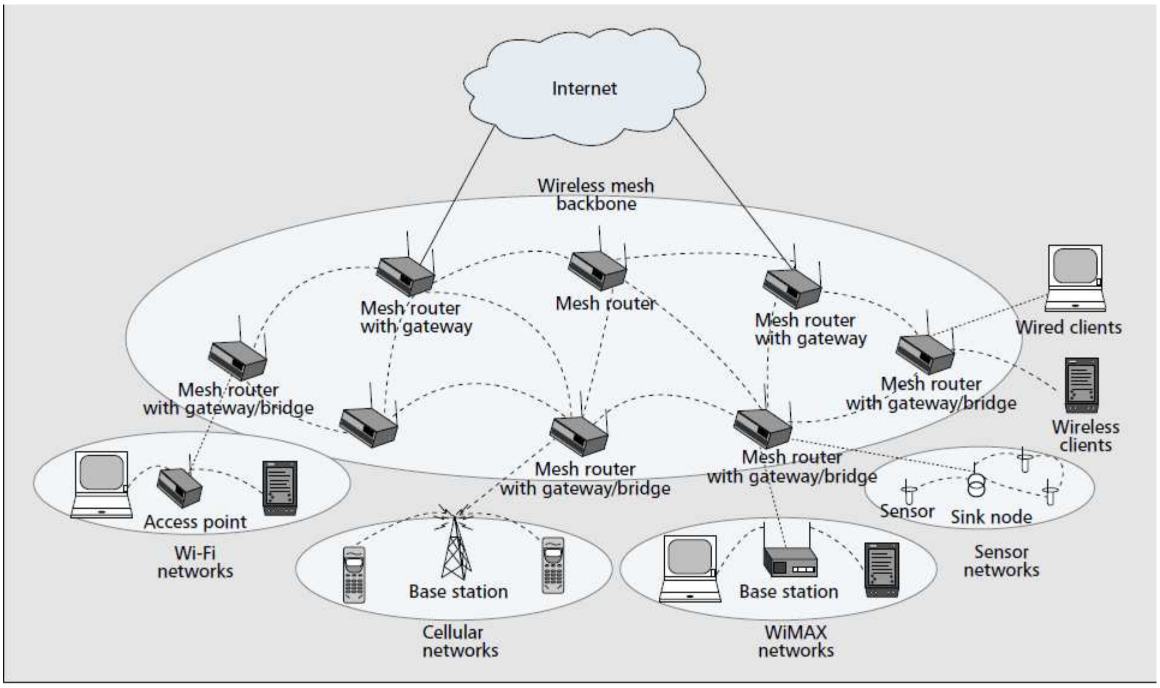

Figure 1.1: Architecture of Infrastructure WMNs. Adapted from [1].

unaware of the backhaul structure and directly connect to one of the wireless mesh routers. With gateway functionality, mesh routers can be connected to the Internet. Such an approach, provides a backbone for conventional clients and also enables integration of WMNs with existing wireless networks, through the gateway/bridge functionalities of mesh routers. Infrastructure WMNs are the most commonly used type. For example, community and neighborhood networks can be built using infrastructure meshing.

- **Client WMNs:** Client meshing provides peer-to-peer networks among client devices (Figure 1.2). In this type of architecture, client nodes constitute the actual network to perform routing and configuration functionalities as well as providing end user applications to customers. Hence, a mesh router is not required for these types of networks. In such networks, the requirements on the end-user devices are increased when compared to infrastructure WMNs, since in Client WMNs, the end-users must perform additional functions such as routing and self configuration.
- **Hybrid WMNs:** Hybrid WMNs are a combination of Infrastructure WMNs and Client WMNs. Mesh clients can access the network through mesh routers as well as directly meshing with other mesh clients.

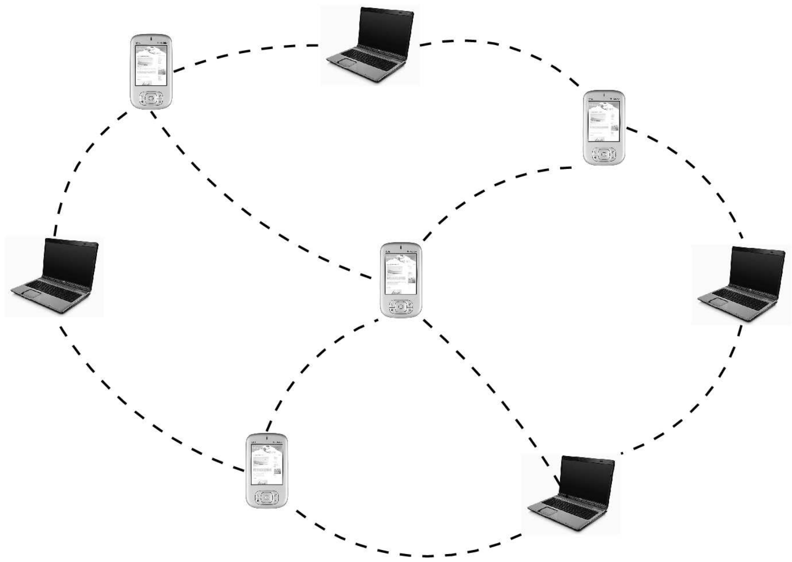

Figure 1.2: Architecture of Client WMNs.

1.1.2 Characteristics of WMNs

The characteristics of WMNs are as follows:

- **Support for Self-configuration and Self-healing:** A WMN is self configurating and does not need any manual configuration. This property makes adding new hardware or changing the location of a wireless mesh router, extremely hassle free. The WMN discovers the new router and automatically adapts to the change. WMNs are also highly resilient. Loss of one or more mesh router, causes other mesh routers to automatically route the messages through alternate mesh routers, thus not disrupting the network's operation. In this way, a mesh network is self-healing because human intervention is not necessary for re-routing of messages.
- **Scalability and Reliability:** A WMN is highly scalable and can handle hundreds or thousands of nodes. As the network's operation doesn't depend on a central control point, adding new mesh routers or gateways is convenient. Reliability, adaptability, and scalability are the most important attributes of a wireless network for commercial deployments.

  Point-to-point networks can provide reliability, but they do not scale to handle more than one pair of end points. Point-to-multipoint networks can handle more end points, but their reliability is determined by the placement of the access point and end points. If environmental conditions result in poor reliability, it is difficult or impossible to adapt a point-to-multipoint network to increase reliability. By contrast, mesh networks are inherently reliable, adapt easily to environmental or architectural constraints, and can scale to handle thousands of end points.

- **Multiple Types of Network Access:** WMNs provide multiple types of network access. In WMNs, both backhaul access to the Internet and peer-to-peer communications are supported. In addition, the integration of WMNs with other wireless networks, and providing services to end-users of these networks, can be accomplished through WMNs.
- **Power Consumption Constraints and Mobility Dependence:** WMNs differ significantly from other multihop wireless networks, like mobile ad-hoc networks and the power constrained wireless sensor networks, in that the mesh networks do not have mobility support and power efficiency as their major issues, as the mesh routers are usually stationary, with power not a constraint.

1.1.3 Application Scenarios

WMNs have the potential to provide support for several important applications, that cannot be supported directly by other wireless networks such as cellular networks, ad hoc networks, and wireless sensor networks. A few of the applications of WMNs are:

- **Broadband Home Networking:** WMNs are well suited for broadband home networking. Currently broadband home networking is realized through IEEE 802.11 WLANs. A major problem with these existing networks, is that meticulous planning is required to eliminate the presence of dead zones (zones without any service coverage). Installing multiple access points, is expensive and inconvenient because of the ethernet wiring of the backhaul network.

  The use of WMNs resolves this issue. By replacing the access points with wireless mesh routers, the communication between the nodes becomes much more flexible and robust to network faults and link failures. Dead zones can be eliminated by adding new mesh routers, or else by changing the locations of the mesh routers. Communication within home networks can be realized through mesh networking without going back to the access hub all the time. Thus, network congestion due to backhaul access can be avoided. On the other hand, Wi-Fis are not capable of supporting ad hoc multi-hop networking.
- **Community and Neighbourhood Networking:** In a community, the common architecture for network access is based on cable or Digital Subscriber

Line (DSL) connected to the Internet, and the last-hop is wireless by connecting a wireless router to a cable or DSL modem. WMNs allow information sharing within the community, without using the Internet. This increases the network resource utilization. The use of WMNs enables many applications such as distributed file storage, distributed file access, and video streaming.

- **Enterprise Networking:** Currently, IEEE 802.11 wireless networks are widely used in various offices to enable wireless networking. However, several wireless networks belonging to different offices in a building or belonging to different buildings are still not connected. Connections among them have to be achieved through wired Ethernet connections, which is the key reason for the high cost of enterprise networks. The addition of backhaul access modems only increases capacity locally, but does not improve the robustness of the network to link failures or network congestion.

  If the access points are replaced by mesh routers, Ethernet wires can be eliminated and multiple backhaul access modems can be shared by all nodes in the entire network, and thus, improve the robustness and resource utilization of the network. Such an architecture is also highly scalable.

- **Other applications:** WMNs can be used for several other purposes. Utilizing WMNs in transportation systems, convenient passenger information services, remote monitoring of in-vehicle security video, and driver communications can be supported. In the area of building automation, WMNs can ease deployment and significantly reduce costs, if the BACnet (Building Automation and Control network) access points are replaced by mesh routers. WMNs also find manifold usage opportunities in health and medical systems, security surveillance systems, metropolitan area networks, and in spontaneous (disaster/emergency) networking and peer-to-peer communications.

1.2 Motivation

A very important consideration for the widespread deployment of infrastructure based broadband WMNs is the support of bandwidth intensive applications such as streaming video and real time voice and video traffic. In order to support such traffic, the network must provide QoS guarantees on the connection. A few of the important aspects to consider while providing such guarantees are 1) the channel assignment algorithm which effectively utilizes and distributes the network resources and 2) the routing protocol which chooses paths that satisfy the QoS requirement. A problem of considerable interest is the estimation of theoretical guarantees that can be provided by any QoS scheme. Such estimates give us clarity on the maximum guarantees that can be provided and also act as a reference with which we can compare the performance of existing protocols.

A major factor which is responsible for the high throughput of WMNs is the usage of multiple interfaces in each mesh router, along with the availability of multiple channels for transmission in the network. Recent studies have shown that the system throughput can further be increased, when both non-overlapped channels and partially overlapped channels are used. It has also been shown in the literature that there is a non-linear increase in system capacity, with the increase in the number of radios at each mesh router. However, due to the limited number of channels available, channel contention is possible leading to collisions and interference. Thus, in order to effectively utilise the multiple radios and multiple channels, efficient channel assignment algorithms are needed.

1.3 Contributions of this Project

The contributions of this project are as follows:

- We analyze the performance in terms of end-to-end call acceptance (which is a measure of the number of calls that can be admitted into the network) of a TDMA based Multi-Channel Multi-Radio (MC-MR) WMN. We consider the case of differentiated class of users and derive upper and lower bounds for the probability of call acceptance. These estimates allow us to answer questions such as the maximum number of high priority calls that can be accepted in the network, and the probability that the network enters into such a state such that no more calls can be accepted. We also study the impact of the number of channels, the number of radios in each mesh router, the channel assignment algorithm, and the routing protocol using these bounds.
- We also introduce and estimate the deterministic guarantee limit which is a mobility independent measure of the number of high-priority calls that can be admitted into the network.
- We jointly study the problem of channel assignment and flow allocation, assuming both non-overlapped and partially overlapped channels are used in the network, such that the aggregate network throughput is maximized or such that the average queueing delay in the network is minimized. We consider

maximizing the aggregate end-to-end communication throughput, rather than sum of link capacities.

- Apart from the standard objective of maximizing network throughput, we show the importance of considering other important factors, such as the average queueing delay in the network, in order to increase the resilience of the network.
- Given the topology of the WMN, we propose a heuristic algorithm for the channel assignment and flow allocation problem in order to scale the solution to bigger networks.

1.4 Organization of this Report

The rest of the report is organized as follows:

Chapter 2: In this chapter, we theoretically analyze the performance in terms of end-to-end call acceptance in TDMA based MC-MR WMNs using queueing theory techniques. We derive bounds for the probability of call acceptance and theoretically estimate the probability of system saturation. We compare the performance of real world routing protocols with the theoretical bounds, and analyze ways to improve the performance of real world routing and channel assignment algorithms.

Chapter 3: In this chapter, we look into the issue of improving the system throughput of WMNs, when both non-overlapped channels and partially overlapped channels are used; by proposing a channel assignment and flow allocation algorithm. In order to scale our algorithm to bigger network topologies, we also propose a polynomially bounded heuristic algorithm.

Chapter 4: This chapter concludes the report and presents some avenues that need to be further explored.

CHAPTER 2

Impact of Multiple Channels and Radios on the Performance of TDMA based Wireless Mesh Networks

2.1 Introduction

Wireless Mesh Networks (WMNs) have recently gained significant attention, since they address important problems like: the problem of growing bandwidth demand along with scarcity of the available spectrum and the need for flexible, easily deployable, self configuring, and adaptable networks. WMNs mix the robustness of cellular networking with the flexibility of ad hoc networking. Emergence of such networks has also been spurred by the development of recent standards such as the IEEE 802.16 [2] and other special working groups such as IEEE 802.11s in the context of IEEE 802.11 networks [3]. WMNs are characterized by an ad hoc backhaul, which is the infrastructure part of the network to which the clients connect. The backhaul, mainly consists of wireless mesh routers that are interconnected in an ad hoc manner. The clients are unaware of the backhaul structure and directly connect to one of the wireless mesh routers.

The main characteristics of fixed WMNs considered here are: 1) There are multiple orthogonal channels of operation available in the network. 2) Mesh nodes are stationary and have multiple radio transceivers, which allow them to communicate simultaneously with more than one neighboring mesh node, using different channels in an interference free manner i.e., a mesh node can be transmitting or receiving in a channel A with mesh node x while simultaneously transmitting or receiving in channel B with mesh node y ($A \neq B$). 3) Due to the limited number of channels available, channel contention is possible, leading to collisions and interference.

In this work, we analyze the performance in terms of end-to-end call acceptance (which is a measure of the number of calls that can be admitted into the network) of a Multi-Channel Multi-Radio (MC-MR) WMN with multimedia traffic (UDP traffic) in terms of the probability of call acceptance (P_{Acc}). We model the system using a Markov Process Model and study the dependence of P_{Acc} on the number of radios in each node ($\mathcal{K}$), the number of channels available in the network (C), and the network load (ρ). We consider a network with calls belonging to different classes based on which the requirements of the calls are prioritized. The QoS constraint of the calls is that of bandwidth. We derive upper (P_{Acc}^{Max}) and lower (P_{Acc}^{Min}) bounds for P_{Acc} and estimate the probability of system saturation (P_{Sat}) which is the probability that no more new calls can be accepted in the network. We also estimate the deterministic guarantee limit which is a measure of the number of high-priority calls that can be admitted into the network. We compare the probability of call acceptance for two routing protocols: Shortest Path (SP) routing protocol and the Multi-Radio Link-Quality Source Routing (MR-LQSR) protocol which uses the Weighted Cumulative Expected Transmission Time (WCETT) metric [4].

2.2 Related Work

MC-MR multi-hop wireless networks have recently received extensive research attention due to their potential future applications. Several studies have been done on the capacity of multi-channel networks in recent times. In [5], the authors analyze a multi-channel network where the number of radios m at each node is less than the number of available channels c. The dependence of the capacity on the ratio $\frac{c}{m}$ is studied. It is shown that in the random network case there are three different capacity regions: 1) the per flow capacity being $\frac{W}{\sqrt{n \log n}}$ (where W is the link bandwidth and n is the number of nodes in the network) when $\frac{c}{m} = O(\log n)$, 2) the per flow capacity being $\Theta(W\sqrt{\frac{m}{nc}})$ when $\frac{c}{m} = \Omega(\log n)$ and also $O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$, and 3) the per flow capacity being $\Theta(\frac{Wm\log\log n}{c\log n})$ when $\frac{c}{m} = \Omega\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$. In [6] and [7], the capacity and connectivity of multi-channel wireless networks with channel switching constraints are considered. In particular, they consider networks where each node is restricted to switch within a set of f channels. Two specific constraint models viz. *adjacent* (c, f) assignment and *random* (c, f) assignment are proposed. In *adjacent* (c, f) channel assignment, a node may switch between f adjacent channels, but the adjacent channel block is randomly assigned. In *random* (c, f) assignment, each node may switch between a pre-assigned random subset of f channels. For the *adjacent* (c, f) assignment case it is shown that when $c = O(\log n)$ the capacity scales as $\Theta\left(W\sqrt{\frac{f}{cn\log n}}\right)$. For the *random* (c, f) assignment case it is initially shown in [6] that the capacity is $O\left(W\sqrt{\frac{p_{rnd}}{n\log n}}\right)$ and also $\Omega\left(W\sqrt{\frac{f}{cn\log n}}\right)$ and later in [7] shown to be $\Theta\left(W\sqrt{\frac{p_{rnd}}{n\log n}}\right)$ where p_{rnd} is the probability that a node can communicate to any other random node in its range. In [8], the authors present efficient schemes to compute maximum throughput and fair bandwidth allocation in multi-radio WMNs.

For the single channel single radio scenario Gupta and Kumar [9] showed that in an arbitrary network, the per flow capacity scales as $\Theta\left(\frac{W}{\sqrt{n}}\right)$ bit-m/s while in a random network it scales as $\Theta\left(\frac{W}{\sqrt{n\log n}}\right)$bits/s. In [10], the authors have analyzed the end-to-end call acceptance and derive theoretical bounds in the single channel ad hoc wireless networks. Several other studies regarding the asymptotic capacity of multi-hop wireless networks [11, 12, 13] have been performed. The authors in [4], present a new routing metric WCETT and a corresponding MR-LQSR protocol to find a high throughput path between a source node and a destination node in multi-radio, multi-hop wireless networks. In [14], the authors have proposed one of the first 802.11 based multi-channel multi-hop WMN architecture. They have developed a set of centralized algorithms for channel assignment, bandwidth allocation, and routing. In a later paper [15] they also present distributed channel assignment and routing algorithms utilizing only local traffic load information. In [16], a distributed channel assignment algorithm, which uses a skeleton assisted channel allocation strategy is proposed. It relies on the construction of a spanning subgraph (called skeleton) of the connectivity graph using a distributed algorithm LMST (Local Minimum Spanning Tree) [17]. The proposed SAFE protocol uses the constructed skeleton to assign channels while preserving connectivity. In [18], the authors consider a fixed MC-MR WMN and propose an MILP based static channel assignment scheme that maximizes the number of bidirectional links that can be activated simultaneously subject to interference constraints.

Most of the previous studies have analyzed the transport capacity of ad hoc and mesh networks for both the MC-MR and the single channel case. In this work, we focus on the end-to-end call acceptance and study its dependence on the network load, routing protocol, the number of radios per node, and the number of available channels. We assume a Time Division Multiple Access (TDMA) based network and investigate the influence of SP routing protocol, MR-LQSR protocol [4], and channel assignment schemes on the end-to-end call acceptance.

2.3 Network Model

We consider a WMN with N mesh nodes randomly dispersed in a terrain of area A. We use the terms mesh node and node interchangeably to refer to the stationary mesh routers that constitute the WMN backbone. There are $\mathcal{K}$ radios in each node and C channels are available for transmission in the region. The interference range of a node j is denoted by $R(j)$. We assume a slotted TDMA mechanism at the MAC layer. Each channel is divided into super-frames and each super-frame consists of B number of time slots over which the nodes in the region transmit and receive packets. In order for a node A to communicate with its neighbor node B, it must reserve one or more slots in the channel that is assigned to the edge A to B. Reuse of slots in a channel is possible subject to interference constraints.

In our analysis, the term call refers to a voice or video session consisting of a stream of packets. A call is said to exist between nodes A and B if there is a set of nodes $(p_0 = A, p_1, \ldots, p_m = B)$ such that p_{k+1} is in the transmission range of p_k and there is a permissible schedule for transmission of packets from node A at each node p_k $(1 \leq k \leq m-1)$. In the absence of preemption, finding such a schedule is equivalent to finding a set of free slots in each of the interference regions $R(p_k)$.

We define the bandwidth of a call as the number of slots used for its transmission. For a call to be set up, we must reserve slots along the multi-hop path of the call. In general, a node may either transmit or receive in a particular slot. A slot is free at a node j $(1 \leq j \leq N)$ in a given channel c $(1 \leq c \leq C)$, if it is neither transmitting nor receiving in that slot. For a node j to transmit in a particular slot in a given channel, the slot must be free at node j and none of the nodes in the interference range of node j $(R(j))$ must be transmitting/receiving in that slot in the given channel. In order for a node j to receive in a particular slot in a given channel, the slot must be free at j and none of the nodes in the interference range $(R(j))$ must be transmitting in that slot in the given channel.

An example of possible transmissions that can exist is shown in Figure 2.1. In the figure, using the above definitions, we can have the following possible scenarios: node 1 transmitting to node 2 in the same channel and the same slot as used by node 4 to transmit to node 3, as nodes 4 and 3 do not hear 1. However, if node 6 transmits to node 5 in the same slot as used by node 4 to transmit to node 3, then node 6 must use a different channel from the one used by node 4, as node 5 hears node 4.

We consider a network $NW = \{1, 2, \ldots, N\}$ of N nodes that can support K classes of calls where class i has a higher priority than class j if $i \leq j$. We allow

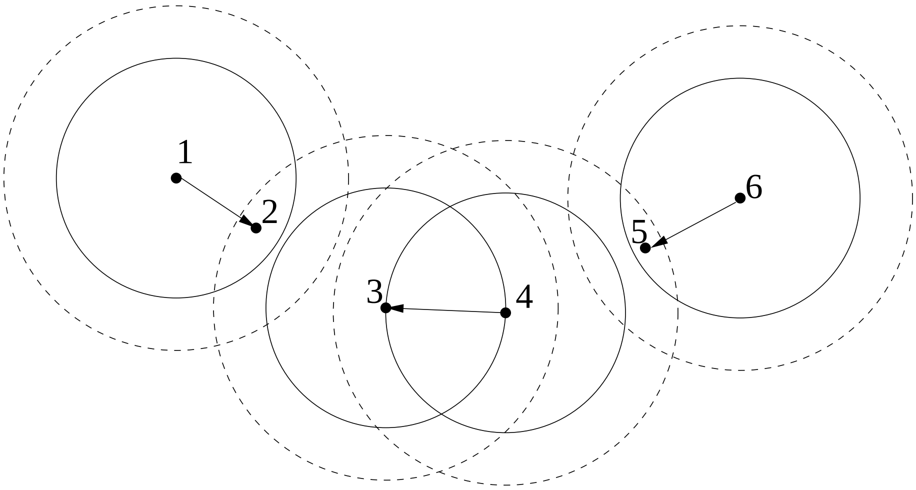

Figure 2.1: An example of possible transmissions. The dashed circles represent the interference range, and the solid circles represent the transmission range.

preemption of calls and estimate the number of calls of various classes that can be supported.

Assumptions for the analysis:

- The arrivals of calls of a class k at each node in the network are *Poisson* distributed with mean λ_k.
- All calls require reservation of a single slot in the super-frame in a given hop. This slot is used for transmitting one packet of the call in each super-frame till the call completes.
- The duration of the call is exponentially distributed with mean duration $\frac{1}{\mu_k}$ secs.
- Nodes are stationary as they constitute the backbone of the WMN.
- The routing algorithm is such that for any path found by the algorithm, the number of nodes on the path that lie within the interference range of any node on the path (inclusive of the node itself) is not greater than some constant ϵ. Failure to have such a constant can lead to a single call utilizing all the slots in the network as shown in Figure 2.2. Each edge is labelled with the slot that is allocated for that hop. Each of the nodes $A, \ldots, G$ is in the interference range of the node j (i.e., $A, \ldots, G \in R(j)$) and also in the interference range of other nodes. Thus slot re-use is not possible in $R(j)$. For example, since the hop B to C is assigned slot number 1(#1) in channel 2, the other hop F to G that is also assigned channel 2 cannot re-use slot number 1(#1) and thus uses another

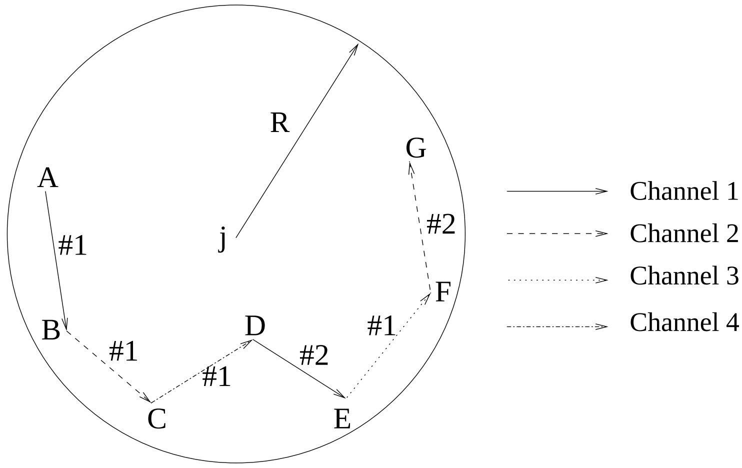

Figure 2.2: A scenario where a single call utilizes several slots in a given region $R(j)$. The edge labels are slot numbers.

slot (#2). We could thus come up with a scenario where a M-hop call uses M slots. We need this constant ϵ to provide bounds on the number of calls that can be accepted.

- We model each node in the network with a distinct Interference Range (IR) and Transmission Range (TR). When $IR > TR$, there may exist nodes that lie outside the transmission range of a node j that can interfere with the communication involving node j. This reduces the spatial reuse of slots. Therefore ϵ is defined in terms of the value of IR/TR based on which the number of slots required for a multi-hop call changes. It can be shown that (Refer 2.3.1 for the proof)

$$\epsilon = \arg\max_{\chi} \left(\frac{\sin\left(\frac{2\pi}{\chi+1}\right)}{\sin\left(\frac{\pi}{\chi+1}\lfloor\frac{\chi+1}{2}\rfloor\right)} \geq \frac{TR(1+\delta)}{IR} \right) \tag{2.1}$$

where δ is a small positive constant. In our analysis, we fix the value of $IR/TR = 2$, based on which we calculate ϵ as 11.

2.3.1 Estimation of the Routing Protocol Constant (ϵ)

In this section, we derive the relation between the Interference Range (IR), Transmission Range (TR), and the parameter ϵ. We note that when $IR > TR$, nodes that lie outside the TR of a node j can still interfere with communications involving node j. This interference leads to a decrease in the spatial re-use of slots, reflected by the increase in the parameter ϵ.

We assume that for every 3 nodes (X, Y, Z) that can hear each other, if a route is to be established from node X to node Z then the direct path X to Z is used and not the multi-hop path of node X to node Z via node Y. Consider a path of χ nodes $(1, 2, \ldots, \chi)$. If appropriate forwarding of the route request packets is done where a node drops all but the first route request that it receives, the above property can be achieved. The distance between two nodes i and j is denoted as $d(i, j)$. We want to place these χ nodes such that

$$\begin{aligned} d(i, i+1) &\leq TR \quad i \in \{1, 2, \ldots, \chi - 1\} \\ TR(1+\delta) \leq d(i, j) &\leq IR \quad j \notin \{i-1, i, i+1\}, i \in \{1, 2, \ldots, \chi\} \\ TR(1+\delta) \leq d(1, \chi) &\leq IR \end{aligned}$$

where δ is a small positive constant and we require $IR > TR(1+\delta)$. The above constraints ensure that the consecutive nodes on the path $(i, i+1), i \in \{1, 2, \ldots, \chi-1\}$ can communicate, the non-consecutive nodes cannot communicate, and that any pair of nodes can interfere with one another.

To construct χ such points, consider a regular polygon of $\chi + 1$ points with the nodes $\{1, 2, \ldots, \chi\}$ mapped to consecutive points of the polygon (except the point $\chi + 1$ which has no mapping). Let R be the radius of the circum-circle of this polygon. Then the distance between consecutive nodes $d(i, i+1) = 2R \sin \frac{\pi}{\chi+1} \leq TR$. The distance between non-consecutive nodes $d(i, j) \geq 2R \sin \frac{2\pi}{\chi+1} \geq TR(1+\delta)$. The maximum distance between any pair of nodes is $2Rsin\left(\frac{\pi}{\chi+1}\lfloor\frac{\chi+1}{2}\rfloor\right) = IR$. Combining these three constraints we have:

$$\frac{sin\left(\frac{\pi}{\chi+1}\right)}{sin\left(\frac{\pi}{\chi+1}\lfloor\frac{\chi+1}{2}\rfloor\right)} \leq \frac{TR}{IR} \leq \frac{1}{1+\delta} \frac{sin\left(\frac{2\pi}{\chi+1}\right)}{sin\left(\frac{\pi}{\chi+1}\lfloor\frac{\chi+1}{2}\rfloor\right)}$$

The maximum value of $\chi+1$ gives us the value of the parameter ϵ for a given value of $\frac{IR}{TR}$. For example, consider a call whose path includes the path from 1 to χ i.e., the call arrives at node 1 from some node $j \notin \{1, 2, \ldots, \chi\}$, follows the path $\{1, 2, \ldots, \chi\}$, and then is transmitted from node χ to some node $\psi \notin \{1, 2, \ldots, \chi - 1\}$. Say node 1 receives the call on slot 1. Each of the links $(i, i+1), i \in \{1, 2, \ldots, \chi - 1\}$ must use a different slot, say $i + 1$, because these nodes interfere with each other. Finally node χ transmits to node ψ on slot $\chi + 1$ due to interference from the remaining nodes in slots $\{1, 2, \ldots, \chi\}$. Thus we have:

$$\epsilon = \arg\max_{\chi} \left(\frac{\sin\left(\frac{2\pi}{\chi+1}\right)}{\sin\left(\frac{\pi}{\chi+1}\lfloor\frac{\chi+1}{2}\rfloor\right)} \geq \frac{TR(1+\delta)}{IR} \right)$$

2.4 Analytical Bounds

In our initial analysis, we assume that call preemption does not occur i.e., a newly arrived high priority call does not preempt a low priority call that is already present in the network. Consider a node j and its interference range $R(j)$. All calls passing through $R(j)$ use up some slots. The calls passing through $R(j)$ consist of calls originating at node j, calls originating from the neighbors of node j, calls originating outside $R(j)$, and either routed through or terminating in $R(j)$.

As described above, calls of a given class arrive at each node according to a Poisson process. However, the distribution of call arrivals in the region $R(j)$ is not Poisson due to the splitting of Poisson streams (non-random forwarding of calls along the available links would mean that the distribution for the region is not Poisson). However, we make use of Kleinrock's Independence Assumption, according to which for moderately heavy call arrival at each node, the net call arrival at the region $R(j)$ can be regarded as Poisson.

Let $\alpha_c(j), 1 \leq c \leq C$ be the fraction of edges in $R(j)$ that are assigned channel c, $f_k(i,j)$ is the fraction of class k calls originating in node i that pass through the region $R(j)$, and $\lambda_{k,c}(j)$ refer to the mean arrival rate of class k calls in $R(j)$ in channel c (i.e., calls entering $R(j)$ in channel c). The mean arrival rate of calls of class k and channel c at $R(j)$ is Poisson distributed with mean:

$$\lambda_{k,c}(j) = \sum_{i \notin R(j)} (\alpha_c(j) f_k(i,j) \lambda_k) + \sum_{i \in R(j)} (\alpha_c(j) \lambda_k) + \alpha_c(j) \lambda_k \tag{2.3}$$

The values of $\alpha_c(j), 1 \leq c \leq C$ are obtained from the network and depends on the number of radios in each node and the channel assignment algorithm. The value of $f_k(i,j)$ depends on the routing protocol. For protocols that lead to heavy loads in the center of the network like SP routing, $f_k(i,j)$ would be high for nodes near the center. We also define $\mu_{k,c}$, the parameter of the exponential distribution for call durations in channel c. Since call times are not affected by the channel, we have $\mu_{k,c} = \mu_k \quad \forall\, 1 \leq c \leq C$ and $\forall\, 1 \leq k \leq K$.

For a single-hop, the state of the system $R(j)$ is given by the number of calls of each class being served in a particular channel. For a single-hop, due to our assumptions about the bandwidth requirement for each call, we can associate each call in the region $R(j)$ (or any hop of a multi-hop call that lies in $R(j)$) with a particular slot in the channel corresponding to that hop. Thus, we can uniquely describe the slot-allocation state i.e., the state of the system based on the number of calls of each class being served in a particular channel. We thus model $R(j)$ as a

KC-dimensional continuous-time Markov process.

$$X(t) = (n_{1,1}, n_{1,2}, \ldots, n_{1,C}, n_{2,1,\ldots}, n_{2,C}, \ldots, n_{k,c}, \ldots, n_{K,C})$$

where $n_{k,c}$ denotes the number of class k calls being served by channel c in $R(j)$ at time t.

The usage of a first-order Markov process to model the system is justified because in the case of wireless networks, the slot allocations at any instant of time are dependent only on the allocations at the previous instant and the rate of arrival and departure of calls at the present instant. To achieve long-term fairness of slot-allocation we could model the system using a process that has a greater dependence on the past. We however, only consider the former scenario here. In our analysis, since we are only interested in the steady state of the process and also because the transitions between the states are restricted, as each state has at most $2KC$ neighboring states and due to the decoupled nature of the processes associated with any given region, we do not have to consider all $\binom{KC+B}{B}$ states that are possible for the most generic Markov process with K classes of calls, with C channels, and B slots. The state explosion for coupled processes or preemptive calls must be handled separately. The interested reader may refer [19] and [20].

We define:

$$P((n'_{1,1}, n'_{1,2}, \ldots, n'_{k,c}, \ldots, n'_{K,C}) \mid (n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C}))$$
$$=$$
$$P(X(t+\Delta t) = (n'_{1,1}, n'_{1,2}, \ldots, n'_{k,c}, \ldots, n'_{K,C}) \mid X(t) = (n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C})$$

as the probability that the system $R(j)$ is in the state $(n'_{1,1}, n'_{1,2}, \ldots, n'_{k,c}, \ldots, n'_{K,C})$ at time $t + \Delta t$ given that it is in the state $(n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C})$ at time t.

We also have:

$$P((n_{1,1}, n_{1,2}, \ldots, n_{k,c}+1, \ldots, n_{K,C}) \mid (n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C})) = \lambda_{c,k}(j)\Delta t \tag{2.4}$$

$$P((n_{1,1}, n_{1,2}, \ldots, n_{k,c}-1, \ldots, n_{K,C}) \mid (n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C})) = n_{k,c}\mu_{c,k}\Delta t \tag{2.5}$$

Note that though the system is slotted, we have modeled the system as a continuous-time Markov Chain instead of using a discrete-time Markov Chain. The approximation that is used here is that the probability of more than one call arriving or departing in a super-frame is low. For small super-frame lengths (as is the case for wireless networks with high bandwidth) this is a valid assumption. Under such an assumption

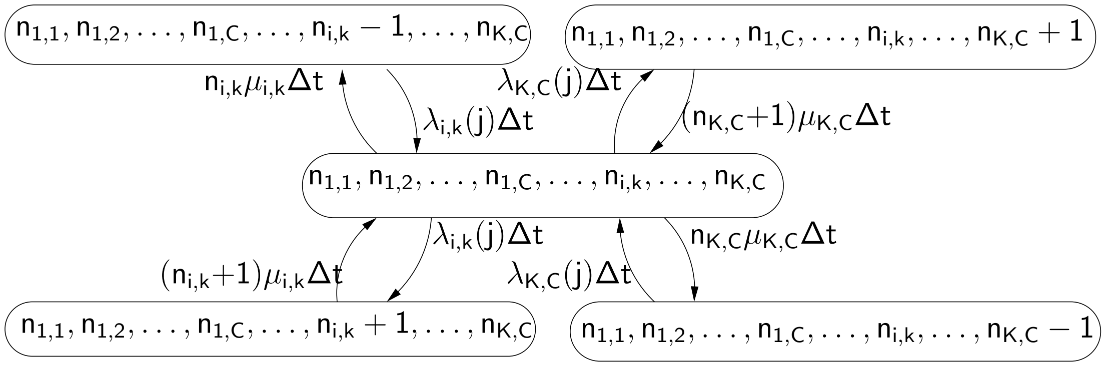

Figure 2.3: The transitions into and out of one of the states of the Markov Process representing the region $R(j)$. The state is $(n_{1,1}, n_{1,2}, \ldots, n_{1,C}, \ldots, n_{i,k}, \ldots, n_{K,C})$, $n_{i,k} > 0 \; \forall \; 1 \leq i \leq K \; and \; \forall \; 1 \leq k \leq C$.

the transition probabilities of the system are similar to those of a continuous-time Markov Chain for a small interval.

The Markov process has a unique steady-state probability distribution. We use Equation (2.4) and Equation (2.5) to get the probability that the system is in a particular state $(n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C})$ as:

$$P(n_{1,1}, n_{1,2}, \ldots, n_{k,c}, \ldots, n_{K,C}) = \frac{1}{G(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!} \tag{2.6a}$$

where $\rho_{k,c}(j) = \frac{\lambda_{k,c}(j)}{\mu_{k,c}}$ and the normalization factor $G(j)$ is given by

$$G(j) = \sum_{\substack{0 \leq \sum_{i=1}^{i=K} n_{i,1} \leq B \\ 0 \leq \sum_{i=1}^{i=K} n_{i,2} \leq B \\ \vdots \\ 0 \leq \sum_{i=1}^{i=K} n_{i,C} \leq B}} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!} \tag{2.6b}$$

We now extend the Markov process to distinguish between calls that terminate in a node in $R(j)$, denoted as type-U calls and those that do not, denoted as type-V calls. We do this because of the unique property of type-V calls: for each new type-V call in $R(j)$, at least one slot that has not already been assigned in the region $R(j)$ must be used. The same is not the case with Type-U calls as slot reuse is possible in some cases. Without loss of generality, we ignore type-U calls in our analysis as it only relaxes the bounds and does not effect the validity of them. Figure 2.4 shows the type-U and type-V calls in the region $R(j)$.

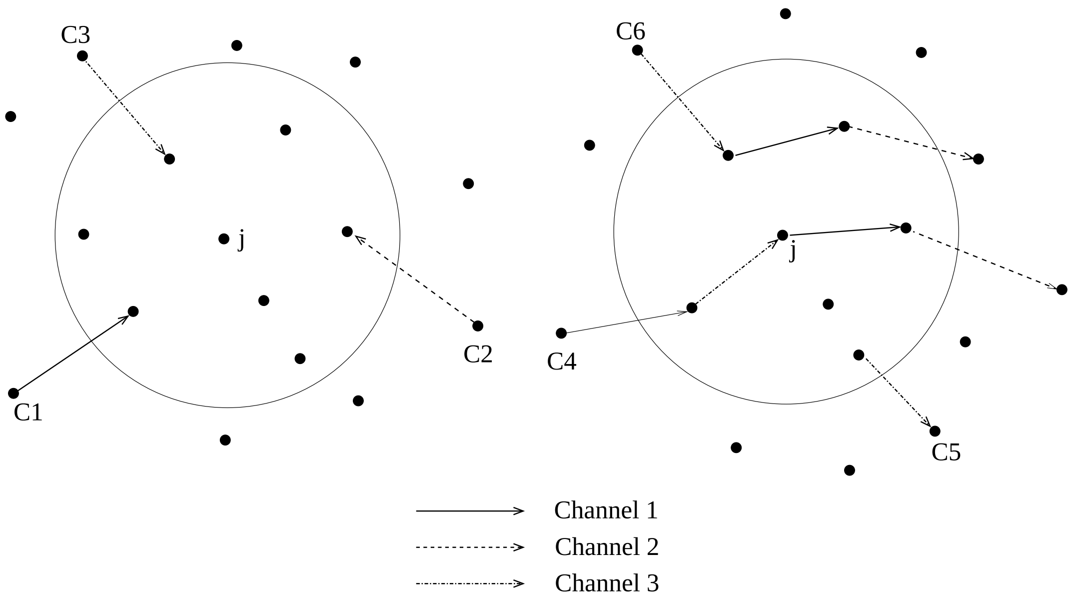

Figure 2.4: Type-U and Type-V calls in the region $R(j)$. Calls $C1, C2, C3$ are type-U calls and $C4, C5, C6$ are type-V calls. For each hop the channels assigned are also shown.

Let f be the fraction of calls that terminate in a node in $R(j)$. Assuming a random destination to be chosen, we have $f = \frac{|N(j)+1|}{N}$, where $N(j)$ denotes the nodes in the interference range of node j. We now define f_c to be the fraction of calls that occur in channel c (note that we are still dealing with only single-hop calls) that terminate in a node in $R(j)$. We have $f_c = f$. (To simplify the analysis, we assume that f_c is unbiased across all channels).

The new state of the system now is

$$(n_{1,1,U}, n_{1,1,V}, n_{1,2,U}, n_{1,2,V}, \ldots, n_{k,c,U}, n_{k,c,V}, \ldots, n_{K,C,U}, n_{K,C,V})$$

where $n_{k,c,U}$ is the number of class k calls in channel c that are of type-U in $R(j)$ and $n_{k,c,V}$ is the number of class k calls in channel c that are of type-V in $R(j)$.

The probability that the system is in the state

$$(n_{1,1,U}, n_{1,1,V}, n_{1,2,U}, n_{1,2,V}, \ldots, n_{k,c,U}, n_{k,c,V}, \ldots, n_{K,C,U}, n_{K,C,V})$$

is

$$P(n_{1,1,U}, n_{1,1,V}, \ldots, n_{k,c,U}, n_{k,c,V}, \ldots, n_{K,C,V}) = \frac{1}{E(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c,U}(j)^{n_{k,c,U}}}{n_{k,c,U}!} \frac{\rho_{k,c,V}(j)^{n_{k,c,V}}}{n_{k,c,V}!} \tag{2.7a}$$

where $\rho_{k,c,U}(j) = \frac{f_c \lambda_{k,c}(j)}{\mu_{k,c}}$, $\rho_{k,c,V}(j) = \frac{(1-f_c)\lambda_{k,c}(j)}{\mu_{k,c}}$ and where the normalization factor $E(j)$ is given by

$$E(j) = \sum_{\substack{0 \le n_{1,1,U}+n_{1,1,V}+n_{2,1,U} \cdots + n_{K,1,V} \le B \\ 0 \le n_{1,2,U}+n_{1,2,V}+n_{2,2,U} \cdots + n_{K,2,V} \le B \\ \vdots \\ 0 \le n_{1,C,U}+n_{1,C,V}+n_{2,C,U} \cdots + n_{K,C,V} \le B}} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c,U}(j)^{n_{k,c,U}}}{n_{k,c,U}!} \frac{\rho_{k,c,V}(j)^{n_{k,c,V}}}{n_{k,c,V}!} \tag{2.7b}$$

Considering only Type-V calls, the probability that the system is in a state $(n_{1,1,V}, n_{1,2,V}, \ldots, n_{k,c,V}, \ldots, n_{K,C,V})$ is given by:

$$P(n_{1,1,V}, \ldots, n_{k,c,V}, \ldots, n_{K,C,V}) = \frac{1}{H(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c,V}(j)^{n_{k,c,V}}}{n_{k,c,V}!} \tag{2.8a}$$

where the normalization factor $H(j)$ is given by

$$H(j) = \sum_{\substack{0 \le \sum_{i=1}^{i=K} n_{i,1,V} \le B \\ 0 \le \sum_{i=1}^{i=K} n_{i,2,V} \le B \\ \vdots \\ 0 \le \sum_{i=1}^{i=K} n_{i,C,V} \le B}} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c,V}(j)^{n_{k,c,V}}}{n_{k,c,V}!}$$

2.4.1 Probability of Call Acceptance for Calls with Fixed Bandwidth

We now derive the probability of call acceptance for both single-hop and multi-hop cases for a non-preemptive system.

Lemma 1: P(Number of used slots in a Region $R(j) \le x$) $\le$ P(Number of type-V calls in $R(j) \le x$), where $x \in \mathbb{N}$.

Proof: As shown in Figure 2.4, for every type-V call, at least one slot that has not already been assigned in the region $R(j)$ must be used. We thus have:

$$\text{Number of used slots in R(j)} \le x$$
$$\Downarrow$$
$$\text{Number of type-V calls in R(j)} \le x$$

Hence
P(Number of used slots in $R(j) \le x$) $\le$ P(Number of type-V calls in $R(j) \le x$)

Lemma 2: P(Number of calls in a Region $R(j) \leq x$) $\leq$ P(Number of used slots in the Region $R(j) \leq \epsilon x$), where $x \in \mathbb{N}$ and ϵ is the routing algorithm dependent constant factor as defined in Section 2.3.

Proof: We have:

$$\text{Number of calls in } R(j) \leq x$$
$$\Downarrow$$
$$\text{Number of used slots in } R(j) \leq \epsilon x$$

Hence P(Number of calls in $R(j) \leq x$) $\leq$ P(Number of used slots in $R(j) \leq \epsilon x$)

2.4.1.1 Theoretical Upper Bound for Probability of Call Acceptance

We now derive the upper bound on the probability of call acceptance for the case of single-hop and multi-hop calls.

Single-Hop Case: Consider a single-hop call from a node j to node i ($i \in N(j)$). Let the channel assigned for edge j to i be c_{ij}. If it is to be unconditionally accepted (i.e., irrespective of the channel it arrives in, $1 \leq c_{ij} \leq C$) then we must have the following for the Probability of Acceptance (P_{Acc})

$$\begin{aligned} &P(A\, single\, hop\, call\, from\, j\, to\, i\;\, in\, channel\, c_{ij}\, is\, accepted) \\ &\quad = P(Number\, of\, free\, slots\, in\, channel\, c_{ij} \geq 1) \\ &\quad = P(Number\, of\, used\, slots\, in\, channel\, c_{ij} \leq B-1) \\ &\quad \leq P(Number\, of\, type-V\, calls\, in\, channel\, c_{ij} \leq B-1) \\ &\quad \leq 1 - P(Number\, of\, type-V\, calls\, in\, channel\, c_{ij} > B-1) \\ &\quad \leq 1 - P(Number\, of\, type-V\, calls\, in\, channel\, c_{ij} = B) \end{aligned}$$

$$\Rightarrow P_{Acc} \leq 1 - \sum_{c=1}^{C} \alpha_c(j) \times P(No.\, of\, type-V\, calls\, in\, channel\, c = B)$$

i.e.,

$$P_{Acc} \leq 1 - \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \leq \sum_{g=1}^{g=K} n_{g,1,V} \leq B \\ \vdots \\ \sum_{g=1}^{g=K} n_{g,i,V} = B \\ \vdots \\ 0 \leq \sum_{g=1}^{g=K} n_{g,C,V} \leq B}} \frac{1}{H(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c,V}(j)^{n_{k,c,V}}}{n_{k,c,V}!}$$

For the case of single-class of calls, in order to have B calls of type-V in a particular channel, say channel 1, there must be no free slots available in channel 1.In this case the above equation reduces to

$$P_{Acc} \leq 1 - \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \leq n_{1,V} \leq B \\ \vdots \\ n_{i,V}=B \\ \vdots \\ 0 \leq n_{C,V} \leq B}} \frac{1}{H'(j)} \prod_{c=1}^{c=C} \frac{\rho_{c,V}(j)^{n_{c,V}}}{n_{c,V}!} \tag{2.9a}$$

where $n_{i,V}$ refers to the number of type-V calls in channel i and the normalization factor $H^{'}(j)$ is

$$H^{'}(j) = \sum_{\substack{0 \leq n_{1,V} \leq B \\ 0 \leq n_{2,V} \leq B \\ \vdots \\ 0 \leq n_{C,V} \leq B}} \prod_{c=1}^{c=C} \frac{\rho_{c,V}(j)^{n_{c,V}}}{n_{c,V}!} \tag{2.9b}$$

Alternatively, consider the set of states for which a single-hop call originating in the system is rejected. For a single class of calls, all calls occurring in channel 1 will be rejected if the system is present in any one of the following set of the states.

$$\{(B,1,0,\ldots,0),\ldots,\underbrace{(B,1,1,\ldots,1)}_{C\ terms}\ldots,(B,B-1,B-1,\ldots,B-1),(B,B,B,\ldots,B)\}$$

P_{Acc} is the probability that the system is not present in any of the above such set of states corresponding to each channel.

Multi-Hop Case: We consider the setup of a M-hop call ($M > 1$) consisting of the nodes $p_1, p_2, \ldots, p_{M+1}$ i.e., the first hop is between the nodes p_1 and p_2 and so on. Let $c_1, c_2, \ldots, c_M$ be the channels assigned along the path for the M-hop call. To establish such a call, we must have a free slot available in each of the hops in the corresponding channel.

So we have:

$$
\begin{aligned}
P_{Acc} = P(&Successfull\ Forwarding\ (SF)\ of\ call\ in\ R(p_1) \times \\
&P(SF\ of\ call\ in\ R(p_2) \mid SF\ of\ call\ in\ R(p_1)) \times \\
&\vdots \\
&P(SF\ of\ call\ in\ R(p_M) \mid SF\ of\ call\ in\ R(p_{M-1}))
\end{aligned} \tag{2.10}
$$

$\Rightarrow$

$$
\begin{aligned}
P_{Acc} = P(&No.\ of\ Free\ Slots\ (FS)\ in\ channel\ c_1\ of\ R(p_1)) \geq 1) \times \\
&P(No.\ of\ FS\ in\ channel\ c_2\ of\ R(p_2) \geq 1 \mid SF\ of\ call\ in\ R(p_1)) \times \\
&\vdots \\
&P(No.\ of\ FS\ in\ channel\ c_M\ of\ R(p_M) \geq 1 \mid SF\ of\ call\ in\ R(p_{M-1}))
\end{aligned} \tag{2.11}
$$

Each of the conditional probabilities in the above Equation (2.11), depends on the exact channel sequence that occurs in the M-hop path from p_1 to p_{M+1}. This will be illustrated in the following example of a 4-hop call. We consider both cases of $C \geq \epsilon$ and $C \leq \epsilon$.

Figure 2.5 shows some of the types of paths to consider while dealing with 4-hop calls with $C \geq \epsilon$. In the following example, we assume $\epsilon = 3$ and $C \geq 3$. The channel assigned to an edge is shown for each hop along with the number of free slots available in that region in parenthesis. In Figure 2.5(a), hop 1 and hop 4 are assigned to the same channel. They do not interfere and hence have B free slots in each of the hops. Note that since $\epsilon = 3$ this is the minimum separation that is required between two hops such that one hop does not interfere with a previous hop assigned to the same channel. In Figure 2.5(b), the first hop (p_1 to p_2) is assigned channel 1 and the number of free slots available in $R(p_1)$ is B. Similarly channel 2 is assigned to the second hop (p_2 to p_3) with B free slots. For the third hop, since $\epsilon = 3$ and channel 1 is already used for the first hop, the number of free slots available for the transmission at $R(p_3)$ decreases by 1. In Figure 2.5(c), since both the first and second hops are assigned channel 1, the number of free slots available for transmission at $R(p_3)$ decreases by 2.

Figure 2.6 shows the case of $C \leq \epsilon$ in which we assume $\epsilon = 4$ and $C \leq 3$. In this case, we note that the maximum number of hops for which we can assign channels that are not already allotted along the path is ϵ. Thus, at $R(p_4)$ the number of free slots available for transmission decreases by 1.

From the above examples, we observe that the exact channel sequence is the determining factor for the various conditional probabilities specified in Equation (2.11).

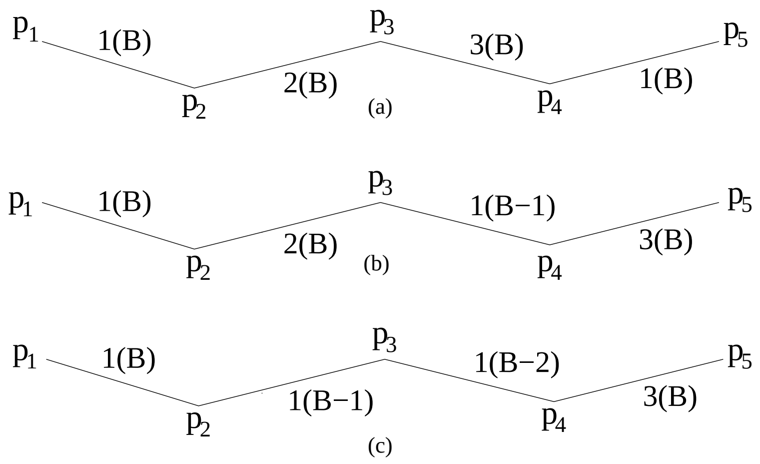

Figure 2.5: Possible paths for $C \geq \epsilon$.

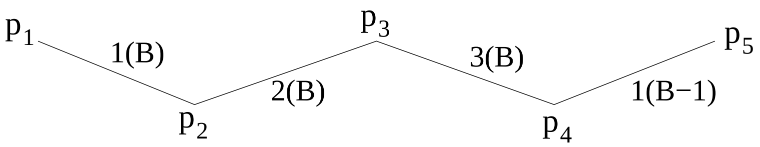

Figure 2.6: A possible path for $C \leq \epsilon$.

This means that getting a generic closed form equation for the upper bound on probability of call acceptance involves considering all possibilities for the channel sequences on the paths and estimating the appropriate values of the conditional probabilities. For higher hop counts, the number of paths to consider become exponential in number. To solve this problem, we make the following assumption, which relaxes the upper bound while providing us with a closed form estimate. We assume the following events to be independent: $Number\ of\ free\ slots\ in\ channel\ c_i\ of\ R(p_i) \geq 1$ and $Successfull\ forwarding\ of\ call\ in\ R(p_{i-1}) \geq 1,\ 1 \leq i \leq M$. This assumption transforms the various conditional probabilities to non-conditional ones. i.e.,

$$P(Number\ of\ FS\ in\ c_i\ of\ R(p_i) \geq 1 \mid SF\ of\ call\ in\ R(p_{i-1})) \tag{2.12}$$
$$=$$
$$P(Number\ of\ FS\ in\ channel\ c_i\ of\ R(p_i) \geq 1)$$

This means that we are neglecting the effect of the slot assignment in the previous hops on the current hop.

For $C \geq \epsilon$, the above assumption holds good if all paths are of the alternating channel (non-interfering type) i.e., paths of the type shown in Figure 2.7 (we assume $\epsilon = 3$ and $C \geq 3$).

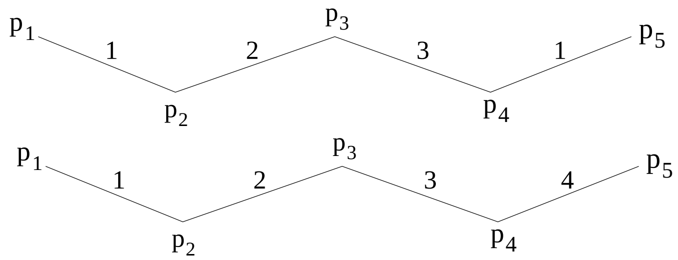

Figure 2.7: Paths with alternating channels (non-interfering) with $\epsilon = 3$ and $C \geq 3$.

For networks without such paths, by making the above assumption we are only overestimating the upper bound (as our assumption causes overestimation of the number of free slots at a node).

For the case of $C \leq \epsilon$, the above assumption only causes a relaxation of the value of the bound (as explained above) and hence the bound derived continues to be an upper bound.

Thus, under the above assumption and using *Lemma* 1 we have

$$\begin{aligned} P_{Acc} \leq\ & P(No.\,of\ type-V\ calls\ in\ channel\ c_1\ of\ R(p_1)) \leq B-1) \times \\ & P(No.\,of\ type-V\ calls\ in\ channel\ c_2\ of\ R(p_2)) \leq B-1) \times \\ & \vdots \\ & P(No.\,of\ type-V\ calls\ in\ channel\ c_M\ of\ R(p_M)) \leq B-1) \end{aligned}$$

from which we have:

$$P_{Acc} \leq \Omega(p_1)\Omega(p_2)\ldots\Omega(p_M) \tag{2.13a}$$

where $\Omega(p_l)$ is the single-hop probability of call acceptance for the hop with source node p_l i.e.,

$$\Omega(p_l) = 1 - \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \leq \sum_{g=1}^{g=K} n_{g,1,V} \leq B \\ \vdots \\ \sum_{g=1}^{g=K} n_{g,i,V} = B \\ \vdots \\ 0 \leq \sum_{g=1}^{g=K} n_{g,C,V} \leq B}} \frac{1}{H(p_l)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c,V}(p_l)^{n_{k,c,V}}}{n_{k,c,V}!}$$

For the case of a single class of calls, we have the following reduced equation

$$P_{Acc} \leq \Omega'(p_1)\Omega'(p_2)\ldots\Omega'(p_M) \tag{2.14a}$$

where

$$\Omega^{'}(p_l) = 1 - \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \le n_{1,V} \le B \\ \vdots \\ n_{i,V}=B \\ \vdots \\ 0 \le n_{C,V} \le B}} \frac{1}{H^{'}(p_l)} \prod_{c=1}^{c=C} \frac{\rho_{c,V}(p_l)^{n_{c,V}}}{n_{c,V}!} \tag{2.14b}$$

In order to get a closed form equation, we replace $\rho_{c,V}(p_j)$, $1 \le j \le M$, $1 \le c \le C$ by $\rho_{c,V}^{Max}$ (the maximum value of $\rho_{c,V}(p_j)$ across all regions). So we have

$$P_{Acc}^{Max} = \left[\Omega^{Max}\right]^M \tag{2.15a}$$

where

$$\Omega^{Max} = 1 - \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \le n_{1,V} \le B \\ \vdots \\ n_{i,V}=B \\ \vdots \\ 0 \le n_{C,V} \le B}} \frac{1}{H^{'}} \prod_{c=1}^{c=C} \frac{\left(\rho_{c,V}^{Max}\right)^{n_{c,V}}}{n_{c,V}!}$$

and

$$H^{'} = \sum_{\substack{0 \le n_{1,V} \le B \\ 0 \le n_{2,V} \le B \\ \vdots \\ 0 \le n_{C,V} \le B}} \prod_{c=1}^{c=C} \frac{\left(\rho_{c,V}^{Max}\right)^{n_{c,V}}}{n_{c,V}!} \tag{2.15b}$$

2.4.1.2 Theoretical Lower Bound for Probability of Call Acceptance

We now consider the lower bound on the probability of call acceptance for single-hop and multi-hop calls.

Single-Hop Case: In this case, for a call to be accepted from node j to its neighbor i, we must have at least one free slot in the channel c_{ij}, the channel assigned to the hop j to i in $R(j)$ i.e.,

$$\begin{aligned}
&P(A\ single\ hop\ call\ from\ j\ to\ i\ \ in\ c_{ij}\ is\ accepted)\\
&\quad = P(No.\ of\ free\ slots\ in\ channel\ c_{ij} \geq 1)\\
&\quad = P(No.\ of\ used\ slots\ in\ channel\ c_{ij} \leq B-1)\\
\Rightarrow P_{Acc} &= \sum_{c=1}^{C} \alpha_c(j) \times P(No.\ of\ used\ slots\ in\ channel\ c \leq B-1)
\end{aligned}$$

Using *Lemma* 2 we have,

$$P_{Acc} \geq \sum_{c=1}^{C} \alpha_c(j) \times P(No.\ of\ calls\ in\ channel\ c \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor)$$

i.e.,

$$P_{Acc} \geq \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \leq n_{1,1}+n_{2,1} \ldots + n_{K,1} \leq B \\ \vdots \\ 0 \leq n_{1,i}+n_{2,i} \ldots + n_{K,i} \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor \\ \vdots \\ 0 \leq n_{1,C}+n_{2,C} \ldots + n_{K,C} \leq B}} \frac{1}{F(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!}$$

where

$$F(j) = \sum_{\substack{0 \leq n_{1,1}+n_{2,1} \ldots + n_{K,1} \leq B \\ 0 \leq n_{1,2}+n_{2,2} \ldots + n_{K,2} \leq B \\ \vdots \\ 0 \leq n_{1,C}+n_{2,C} \ldots + n_{K,C} \leq B}} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!}$$

Multi-Hop Case: Consider the setup of a M-hop call ($M > 1$) along the nodes $(p_1, p_2, \ldots, p_{M+1})$. The probability of call acceptance is determined as follows:

$$\begin{aligned}
P_{Acc} \geq\ & P(Number\ of\ calls\ in\ R(p_1) \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor) \times\\
& P(Number\ of\ calls\ in\ R(p_2) \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor) \times\\
& \vdots\\
& P(Number\ of\ calls\ in\ R(p_{M-1}) \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor)
\end{aligned}$$

which gives

$$P_{Acc} \geq \Phi(p_1)\Phi(p_2)\ldots\Phi(p_M) \tag{2.17}$$

where

$$\Phi(j) = \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \leq n_{1,1}+n_{2,1}\ldots+n_{K,1} \leq B \\ \vdots \\ 0 \leq n_{1,i}+n_{2,i}\ldots+n_{K,i} \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor \\ \vdots \\ 0 \leq n_{1,C}+n_{2,C}\ldots+n_{K,C} \leq B}} \frac{1}{F(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!}$$

and

$$F(j) = \sum_{\substack{0 \leq n_{1,1}+n_{2,1}\ldots+n_{K,1} \leq B \\ 0 \leq n_{1,2}+n_{2,2}\ldots+n_{K,2} \leq B \\ \vdots \\ 0 \leq n_{1,C}+n_{2,C}\ldots+n_{K,C} \leq B}} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!}$$

For the case of a single class of calls the above equations reduce to

$$P_{Acc} \geq \Phi'(p_1)\Phi'(p_2)\ldots\Phi'(p_M) \tag{2.18}$$

where

$$\Phi'(j) = \textstyle\sum_{i=1}^{i=C} \alpha_i(j) \times \displaystyle\sum_{\substack{0 \leq n_{1,1} \leq B \\ \vdots \\ 0 \leq n_{1,i} \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor \\ \vdots \\ 0 \leq n_{1,C} \leq B}} \frac{1}{F'(j)} \prod_{c=1}^{c=C} \frac{\rho_{1,c}(j)^{n_{1,c}}}{n_{1,c}!}$$

and

$$F'(j) = \sum_{\substack{0 \leq n_{1,1} \leq B \\ 0 \leq n_{1,2} \leq B \\ \vdots \\ 0 \leq n_{1,C} \leq B}} \prod_{c=1}^{c=C} \frac{\rho_{1,c}(j)^{n_{1,c}}}{n_{1,c}!}$$

Replacing $\rho_{1,c}(p_j)$, $1 \leq j \leq M$, $1 \leq c \leq C$ by $\rho_{1,c}^{Min}$ (the minimum value of $\rho_{1,c}(p_j)$ across all the regions) we have

$$P_{Acc}^{Min} = \left(\Phi^{Min}\right)^M \tag{2.19}$$

where

$$\Phi^{Min} = \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0 \leq n_{1,1} \leq B \\ \vdots \\ 0 \leq n_{1,i} \leq \left\lfloor \frac{B-1}{\epsilon} \right\rfloor \\ \vdots \\ 0 \leq n_{1,C} \leq B}} \frac{1}{F"} \prod_{c=1}^{c=C} \frac{\left(\rho_{1,c}^{Min}\right)^{n_{1,c}}}{n_{1,c}!}$$

and

$$F" = \sum_{\substack{0 \leq n_{1,1} \leq B \\ 0 \leq n_{1,2} \leq B \\ \vdots \\ 0 \leq n_{1,C} \leq B}} \prod_{c=1}^{c=C} \frac{\left(\rho_{1,c}^{Min}\right)^{n_{1,c}}}{n_{1,c}!}$$

2.4.1.3 Probability of System Saturation

We now analyze the probability of system saturation (P_{Sat}) for single-hop calls. For the system to be in a saturated state i.e., in a state where no more calls can be accepted, we must have all the slots in each channel used completely i.e., the maximum number of type-V calls is B in each channel. So we have

$$P_{Sat}(R(j)) = \prod_{1 \leq c \leq C} P(B \; slots \; are \; used \; in \; channel \; c) \tag{2.20}$$

i.e.

$$P_{Sat}(R(j)) = \sum_{\substack{n_{1,1}+n_{2,1} \ldots + n_{K,1} = B \\ \vdots \\ n_{1,i}+n_{2,i} \ldots + n_{K,i} = B \\ \vdots \\ n_{1,C}+n_{2,C} \ldots + n_{K,C} = B}} \frac{1}{K(j)} \prod_{k=1}^{k=K} \prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!} \tag{2.21}$$

where

$$K(j) \quad = \sum_{\substack{0\le n_{1,1}+n_{2,1}\ldots+n_{K,1}\le B \\ 0\le n_{1,2}+n_{2,2}\ldots+n_{K,2}\le B \\ \vdots \\ 0\le n_{1,C}+n_{2,C}\ldots+n_{K,C}\le B}} \prod_{k=1}^{k=K}\prod_{c=1}^{c=C} \frac{\rho_{k,c}(j)^{n_{k,c}}}{n_{k,c}!}$$

2.4.2 Probability of Call Acceptance for Calls with Varying Bandwidth

We extend the above results to the case of calls with varying bandwidth requirements. The bandwidth of a call depends on the class it belongs to. We consider the case where all calls are of equal priority (there is no call preemption in the network). We assume that calls of class i require i slots. Since there is a variation in the number of slots for each call, *Lemma 1* and *Lemma 2* change. The modified lemmas are:

Lemma 3: P(Number of used slots in a Region $R(j) \le x) \le$ P($\sum_{i=1}^{i=K} i \times$ Number of type-V calls of class i in $R(j) \le x$), where $x \in \mathbb{N}$.

Lemma 4: $P(\sum_{i=1}^{i=K} i \times$ Number of calls of class i in a Region $R(j) \le x) \le$ P(Number of used slots in the Region $R(j) \le \epsilon x$), where $x \in \mathbb{N}$ and ϵ is the routing algorithm dependent constant factor as defined in Section 2.3.

Using these Lemmas, we have the following bounds on the probability of call acceptance for the call of class b:

2.4.2.1 Theoretical Upper Bound for Probability of Call Acceptance

For the single-hop case, we have the upper bound as:

$$P_{Acc}^{Max}(b) \quad = 1 - \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0\le\sum_{m=1}^{K} m\cdot n_{m,1,V}\le B \\ \vdots \\ B-b+1\le\sum_{m=1}^{K} m\cdot n_{m,i,V}\le B \\ \vdots \\ 0\le\sum_{m=1}^{K} m\cdot n_{m,C,V}\le B}} \frac{1}{\mathcal{H}} \prod_{k=1}^{k=K}\prod_{c=1}^{c=C} \frac{\left(\rho_{k,c,V}^{Max}\right)^{n_{k,c,V}}}{n_{k,c,V}!} \quad (2.22)$$

where

$$\mathcal{H} = \sum_{\substack{0\le\sum_{m=1}^{K} m\cdot n_{m,1,V}\le B \\ 0\le\sum_{m=1}^{K} m\cdot n_{m,2,V}\le B \\ \vdots \\ 0\le\sum_{m=1}^{K} m\cdot n_{m,C,V}\le B}} \prod_{k=1}^{k=K}\prod_{c=1}^{c=C} \frac{\left(\rho_{k,c,V}^{Max}\right)^{n_{k,c,V}}}{n_{k,c,V}!}$$

For the multi-hop case (for a M-hop call), we have the upper bound as:

$$P_{Acc}(b) \le \Theta(p_1, b)\Theta(p_2, b)\ldots\Theta(p_M, b)$$

where $\Theta(p_i, b)$ is the single-hop probability of call acceptance for calls of class b with source node p_i.

2.4.2.2 Theoretical Lower Bound for Probability of Call Acceptance

We have the following for the lower bound:

$$P_{Acc}(b) \ge \sum_{i=1}^{i=C} \alpha_i(j) \times \sum_{\substack{0\le\sum_{m=1}^{K} m\cdot n_{m,1}\le B \\ \vdots \\ 0\le\sum_{m=1}^{K} m\cdot n_{m,i}\le \left\lfloor \frac{B-b}{\epsilon} \right\rfloor \\ \vdots \\ 0\le\sum_{m=1}^{K} m\cdot n_{m,C}\le B}} \frac{1}{\mathcal{F}} \prod_{k=1}^{k=K}\prod_{c=1}^{c=C} \frac{(\rho_{k,c}(j))^{n_{k,c}}}{n_{k,c}!}$$

where $\mathcal{F}$ is the normalization factor.

For the multi-hop case (for a M-hop call), we have the lower bound as:

$$P_{Acc}(b) \ge \Psi(p_1, b)\Psi(p_2, b)\ldots\Psi(p_M, b)$$

where $\Psi(p_i, b)$ is the single-hop probability of call acceptance for calls of class b with source node p_i.

2.4.3 Deterministic Guarantees

To guarantee the acceptance of certain calls in the network, we look into the issue of providing deterministic guarantees. This guarantee can be made possible by the use of a system of call classes, sub-classes, and priority. A rank-based priority scheme (prioritizing the calls depending on the class to which they belong) is used to prioritize the calls, which is necessary if we are to provide guarantees for the call acceptance. Based on the address or the ID of source, calls belonging to the highest priority are assigned to sub-classes. Call Admission ensures that only one call belonging to a sub-class can exist in the system. This implies that a node can originate only one high priority call. We make these assumptions and derive the appropriate bounds for deterministic guarantees for the case of an MC-MR WMN.

2.4.3.1 Deterministic Guarantee Limit

We define the deterministic guarantee limit $\mathcal{D}$ as the number of sub-classes of the highest priority class that can be ensured deterministic call acceptance. We call these sub-classes deterministic sub-classes and the other sub-classes as probabilistic sub-classes. Using *Lemma* 2, we have the following:

Number of calls in $R(j)$ in channel c is $x \Rightarrow$ Number of used slots in channel c in $R(j) \leq \epsilon x$

Now if we choose $x = \left\lfloor \frac{B}{\epsilon} \right\rfloor$ for each channel c $(1 \leq c \leq C)$ then we have the number of used slots in each channel in $R(j) \leq B$. If we have a total of $C \times \left\lfloor \frac{B}{\epsilon} \right\rfloor$ sub-classes with $\left\lfloor \frac{B}{\epsilon} \right\rfloor$ sub-classes in each channel, then $\forall j$ $(1 \leq j \leq N)$, we will have, number of used slots in $R(j)$ in each channel c $(1 \leq c \leq C) \leq B$. Hence we observe that this is the number of sub-classes that can be definitely accepted by all the regions of the network at any given time. We can achieve this by allocating a unique set of slots for each sub-class from its corresponding channel. Thus we have the deterministic limit $\mathcal{D} \geq C \times \left\lfloor \frac{B}{\epsilon} \right\rfloor$. Note that this is a lower bound on the value of $\mathcal{D}$.

2.5 Theoretical Studies

In this section, we analyze the dependence of the end-to-end call acceptance on $\mathcal{K}$, C, ρ, and the channel assignment algorithm.

2.5.1 Impact of the Channel Assignment Algorithm on the Probability of Call Acceptance

We now analyze the behavior of the upper bound on the probability of call acceptance (P_{Acc}^{Max}) under different channel assignment schemes with varying load (ρ). In particular, we consider a network with $\mathcal{K} = 2$ and $C = 3$ and study the behavior of P_{Acc}^{Max}

under different channel assignment schemes. We note that the channel assignment scheme decides the values of $\alpha_1(j)$, $\alpha_2(j)$, and $\alpha_3(j)$, $1 \leq j \leq N$, and hence the upper bound.

From Figure 2.8, we can see that a channel assignment scheme that utilizes all the available channels in a balanced manner has the highest value of P_{Acc}^{Max}. This is in accordance with the observation that in any network, usage of all the available channels in a balanced fashion leads to a better resource utilization and hence a higher probability of call acceptance. In case of unbalanced usage, the contention in the heavily loaded channel increases and in the lightly loaded channel decreases. The high contention can be alleviated by shifting some traffic from the heavily loaded channel to the lightly loaded channel.

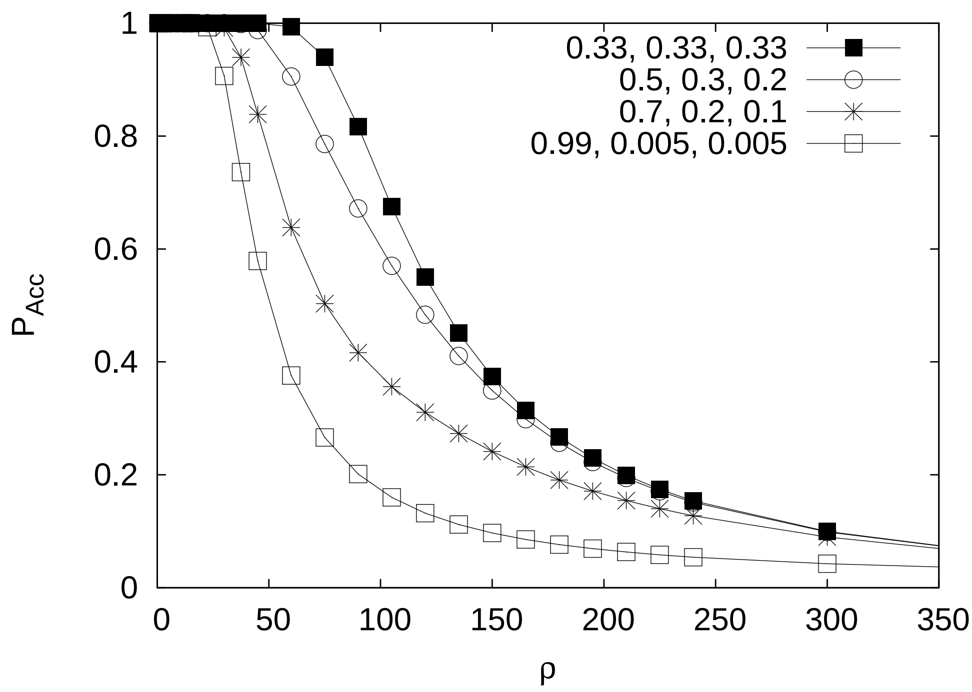

Figure 2.8: Dependence of P_{Acc}^{Max} on the channel assignment scheme for $\mathcal{K} = 2$ and $C = 3$ for different values of $\alpha_1, \alpha_2, \alpha_3$.

2.5.2 Impact of the Number of Channels on the Probability of Call Acceptance

Figure 2.9 shows the dependence of the probability of call acceptance on the number of channels with varying load. In this figure, we have considered the ideal bounds i.e., equal utilization of all channels (by taking all α_i's equal). From the figure, we

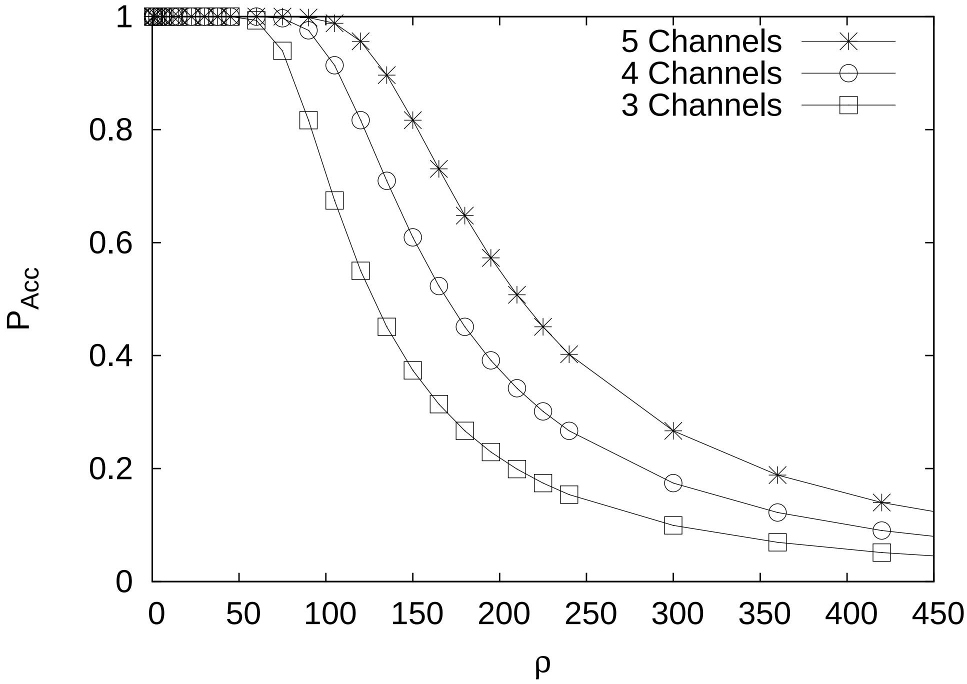

Figure 2.9: Dependence of P_{Acc}^{Max} on the number of channels for $\mathcal{K} = 2$.

see that as the number of channels increases, the upper bound also increases for a fixed number of radios in each node ($\mathcal{K} = 2$).

2.5.3 Impact of the Number of Radios on the Probability of Call Acceptance

Figure 2.10 shows the dependence of the probability of call acceptance on the number of radios for a fixed number of channels in the region ($C = 5$) with varying load. A change in the number of radios causes the channel assignment algorithm to change the α_c $(1 < c < C)$ values (the α_c values are obtained from the channel assignment algorithm), which in turn affects the upper bound. The values of α_c $(1 < c < C)$ are obtained using the distributed channel assignment algorithm Safe [16] for various number of radios, and the upper bound is compared against the ideal value, which corresponds to a completely balanced channel utilization (i.e., all α_i's are equal). From the results, we see that an increase in the number of radios leads to an increase in the probability of call acceptance. Note that the curves for the $\mathcal{K} = 3$ and $\mathcal{K} = 4$ case overlap with one another. This just reflects the performance of the channel assignment algorithm, because in spite of an increase in the number of radios available in the network, the channel assignment algorithm is unable to provide a significantly better channel assignment.

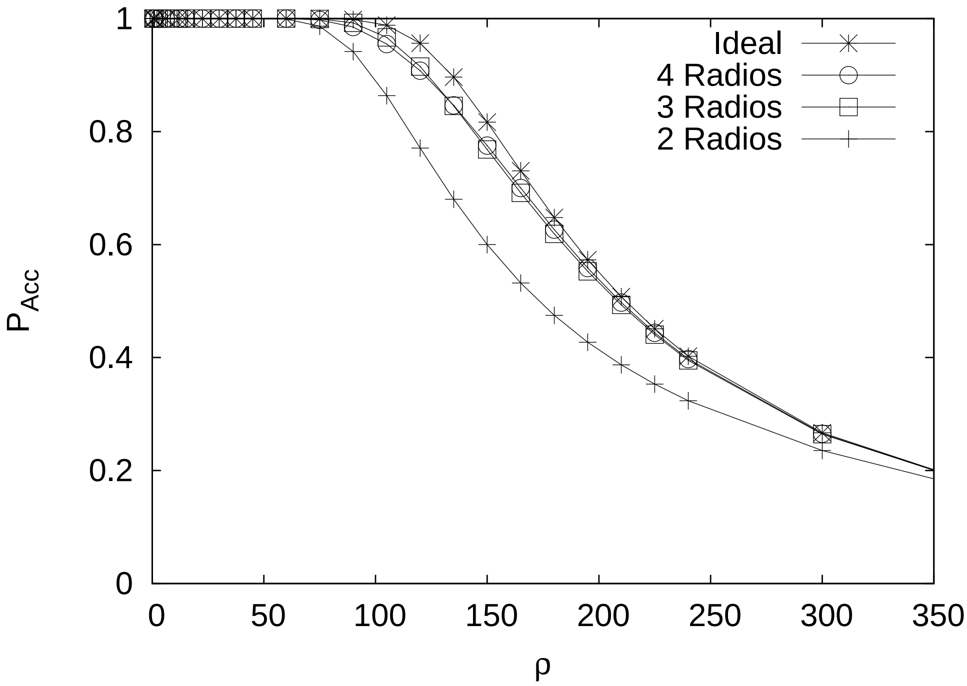

Figure 2.10: Dependence of P_{Acc}^{Max} on the number of radios and the channel assignment algorithm for $C = 5$.

2.5.4 Probability of System Saturation

The variation of the probability of system saturation for single-hop calls is shown in Figure 2.11. From the figure, we can observe that P_{Sat} is nearly zero for moderate to high loads and takes on a significant value at very high loads. Thus during normal state of operation, the network rarely enters into a state of saturation i.e., it is always possible to ensure acceptance for some fraction of calls during low and moderate loads.

2.6 Simulation Studies

We developed a network simulator written in C++ that simulates the TDMA slot allocation in an MC-MR WMN. Calls are generated at each node based on a Poisson process and the call durations are exponentially varied. In order for a call arriving at a particular node to be successful, two conditions must be satisfied. Initially, a path must exist between the source and the destination nodes (where the destination is chosen randomly from the other nodes for each call) of the call. Secondly, along each

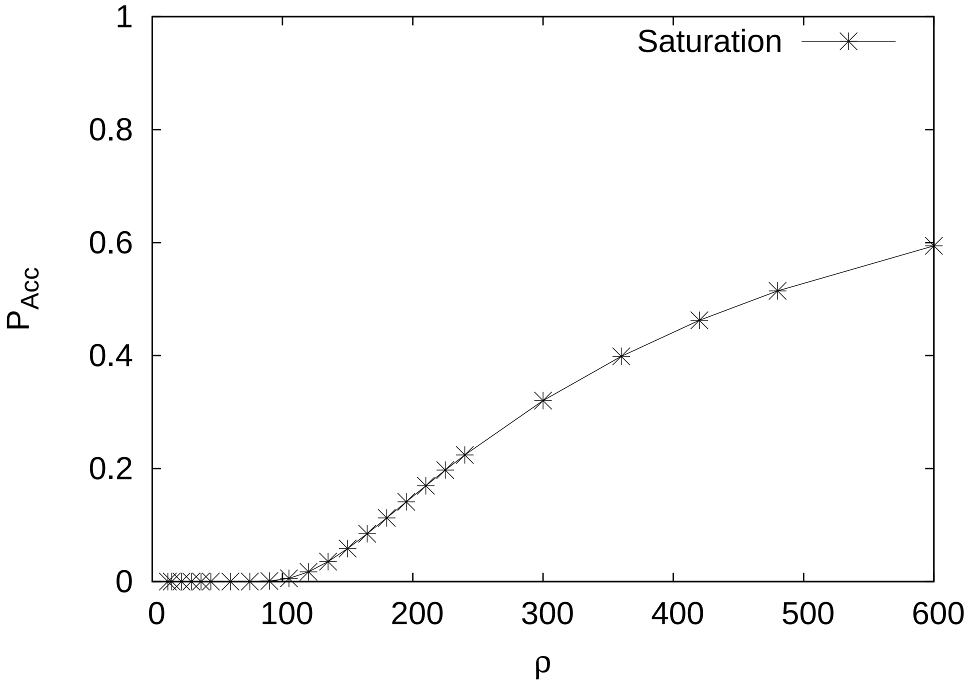

Figure 2.11: P_{Sat} vs ρ for 1-hop calls with $\mathcal{K} = 2$ and $C = 3$.

node in the chosen path, there must be a free slot available (slot allocation is done in a greedy manner) i.e., the call is dropped if the number of free slots is found to be inadequate at an intermediate node.

The simulation parameters are as shown in Table 2.1. The number of nodes N in the network is 75. The number of slots B in each channel is 32. The area of the network under consideration is $1500m \times 1500m$ in which the nodes are randomly dispersed. Transmission range of a node is $300m$. The interference range is double the transmission range i.e., $600m$. The value of $\epsilon = 11$. The number of radios in each node and the number of channels available in the network are varied for each simulation as described in the next section. The simulation runs are carried out for 20 seeds and the duration of each simulation run is 400 secs. The simulation parameters are chosen such that the average number of neighbors for each node is quite high (around 8) in order to simulate a real world dense mesh network.

To study the variation of probability of call acceptance with the network load ρ, we vary the call arrival rate at each node to get $\rho = AverageCallArrivalRate\ (\lambda_{avg}) \times AverageCallDuration\ (\frac{1}{\mu_{avg}})$. We then study the probability of call acceptance under two routing protocols: SP and MR-LQSR. For both cases, to compare the simulation results with the theoretical bounds, we need to translate the ρ value to the $\rho_{c,V}^{Max}$ value. This is done using Equation (2.3) by measuring the average fraction of calls

Table 2.1: Simulation parameters.

Parameter	Value
N	75
B	32
Network Area	$1500m \times 1500m$
TR	$300\,m$
IR	$600\,m$
ϵ	11
Average Call Duration	$30\,s$
Number of Seeds	20
Simulation Time	$400\,s$

that pass through the region for each channel which depends on the routing protocol. We measure the probability of call acceptance for calls of various hop counts.

2.6.1 Impact of the Routing Protocols on the Probability of Call Acceptance

In this section, we analyze the dependence of the end-to-end call acceptance on $\mathcal{K}$, C, ρ, and the routing protocol. For our analysis, we use a distributed channel assignment algorithm given in [16].

2.6.1.1 Routing Protocols

The details of the routing protocols compared in our analysis are explained below.

Shortest Path Routing In this routing protocol, the shortest path (using the Euclidean distance as the measure of distance) between the source and destination is chosen as the routing path. In [21], the authors prove that in a dense network the average path length obtained is $0.905R$ where R is the radius of the network. This leads to heavy loads at the center of the network. Due to this, there is a formation of hotspots at the center that decreases the probability of call acceptance.

Multi Radio-Link Quality Source Routing (MR-LQSR) In [4], the authors propose a WCETT metric and a routing protocol MR-LQSR, which chooses high throughput paths between the source and destination. Based on the Expected Transmission Time (ETT) of a packet over a link, each link is assigned a weight. The ETT depends on the loss rate and the bandwidth of link. The individual link weights are combined into a path metric called WCETT that explicitly accounts for the interference among links that use the same channel. Among the various paths between the source and destination, the one with the lowest WCETT value is chosen as the routing path.

In our analysis we relate the loss rate of a link with the value of the signal strength at the end of a given link. The variation of signal strength ($P_{dBm}(d)$) with distance (d) and the shadowing (ε) is given by([22])

$$P_{dBm}(d) = P_{dBm}(d_0) - 10\alpha \log_{10}\left(\frac{d}{d_0}\right) + \varepsilon$$

where $P_{dBm}(d_0)$ is the signal strength at reference distance (d_0) and where α is the environment pathloss exponent.

In [23] the authors have analyzed a measurement driven deployment strategy and a data driven model to study the impact of design and topology decisions on network performance. In our analysis we use the values used in [23] which were derived from actual physical measurements on a reference real world network. In particular we have $\alpha = 0.33$ and ε which is zero-mean Gaussian Random Variable with standard deviation ($\sigma_\epsilon = 5.9$). The link strength is represented by a random variable X and the probability that the signal strength is above a certain minimum value T_{Min} at a distance d is computed using the Q-function (complement of the CDF of the Standard Gaussian) to calculate the Gaussian tail probability. We can consider this probability to be a measure of the reliability of the link ($p_{reliability}$).

$$p_{reliability} = P(X > T_{Min}) = Q\left(\frac{T_{Min} - P_{dBm}(d)}{\sigma_\epsilon}\right) \tag{2.23a}$$

$$\epsilon \sim N(0, \sigma_\epsilon^2) \tag{2.23b}$$

where $T_{Min} = -75dBm$ is the minimum acceptable average signal strength required for successful transmission over a link. We consider symmetric links and hence have the same values for the forward (p_f) and backward (p_b) link loss probability, each being equal to $(1 - p_{reliability})$. We finally have the ETT of a link from [4] as

$$ETT = \frac{S}{B} \times \frac{1}{(1-p)}$$

where S is the packet size and B the link bandwidth. p is the probability that a packet transmission is not successful. We have $p = 1 - (1 - p_f) \times (1 - p_b)$ and hence the above equation becomes

$$ETT = \frac{S}{B} \times \frac{1}{(p_{reliability})^2} \tag{2.24}$$

Considering a n-hop path in a system with k channels, we have the WCETT metric [4] as:

$$WCETT = (1 - \beta) \times \sum_{i=i}^{n} ETT_i + \beta \times \max_{1 \leq j \leq k} X_j \tag{2.25}$$

where ETT_i is the ETT for hop-i, X_j is the sum of transmission times of hops on channel j, and β is a tunable parameter subject to $0 \leq \beta \leq 1$. We use $\beta = 0.5$ for our simulation as done in [4].

We assume a constant bandwidth for each link to calculate the ETT. We note that, for a single-hop, due to the above method employed for ETT estimation, the MR-LQSR protocol reduces to the SP routing protocol (i.e., ETT of a link is only dependent on the length of the link). Thus, the MR-LQSR protocol is considered only for multi-hop calls.

2.6.1.2 Probability of Call Acceptance

We study the probability of call acceptance of SP routing and MR-LQSR [4] and compare them with the theoretical bounds at different loads. These studies have been repeated for hopcount values of one, two, and three.

Figures $2.12 - 2.20$, represent the variation of probability of call acceptance for a network with varying C, $\mathcal{K}$, and hop-counts. As the hop count of the calls increases, we observe a decrease in the theoretical bound as well as the probability of call acceptance under SP routing and MR-LQSR protocols, as expected.

In Figures $2.12 - 2.17$, with an increase in the number of channels (from 3 to 4), we observe an increase in both the theoretical upper bound and probability of call acceptance under SP and MR-LQSR. This is due to the fact that with an increase in the number of channels, paths with more diverse channels are possible decreasing interference among the nodes in the region. This increase is observed across all hop counts. The trend of decreasing in probability of call acceptance for calls of higher hop counts is also observed here.

In Figures $2.15 - 2.20$, with an increase in the number of radios from 2 to 3, we observe an increase in the theoretical upper bound as well as the probability of call acceptance under SP and MR-LQSR. The increase in the number of radios leads to a more diversified availability and subsequent usage of channels at each node (this

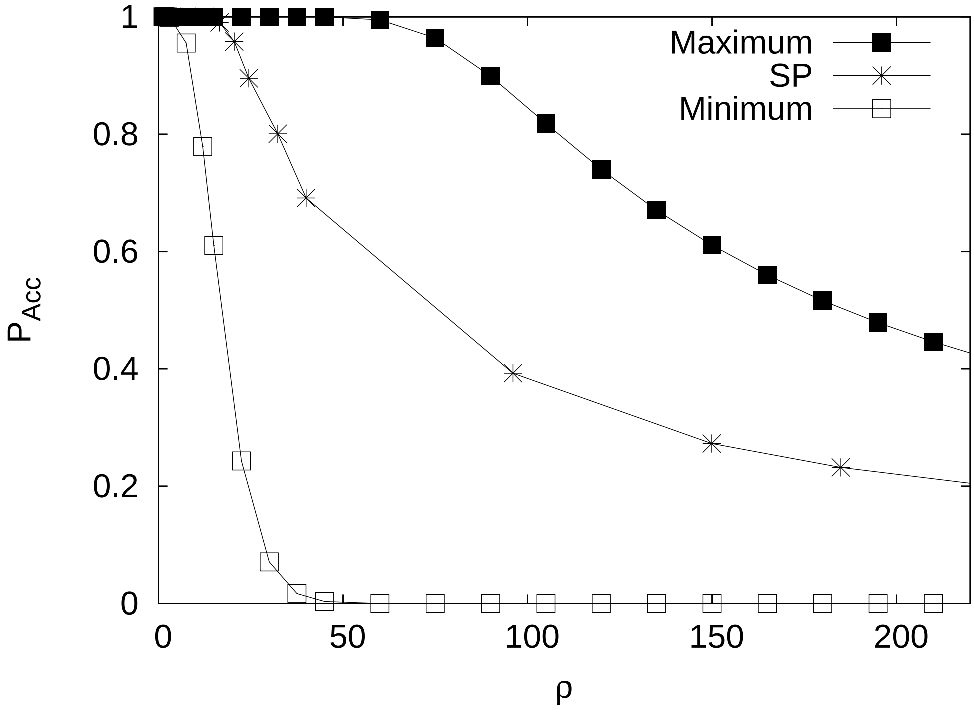

Figure 2.12: P_{Acc} vs ρ for 1-hop calls with $\mathcal{K} = 2$ and $C = 3$.

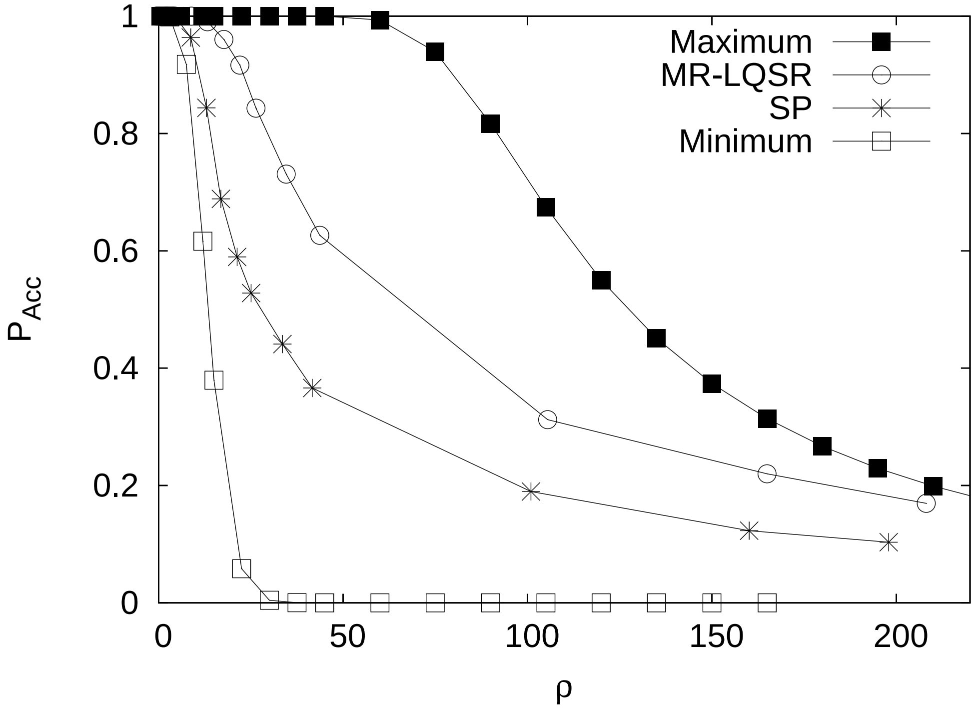

Figure 2.13: P_{Acc} vs ρ for 2-hop calls with $\mathcal{K} = 2$ and $C = 3$.

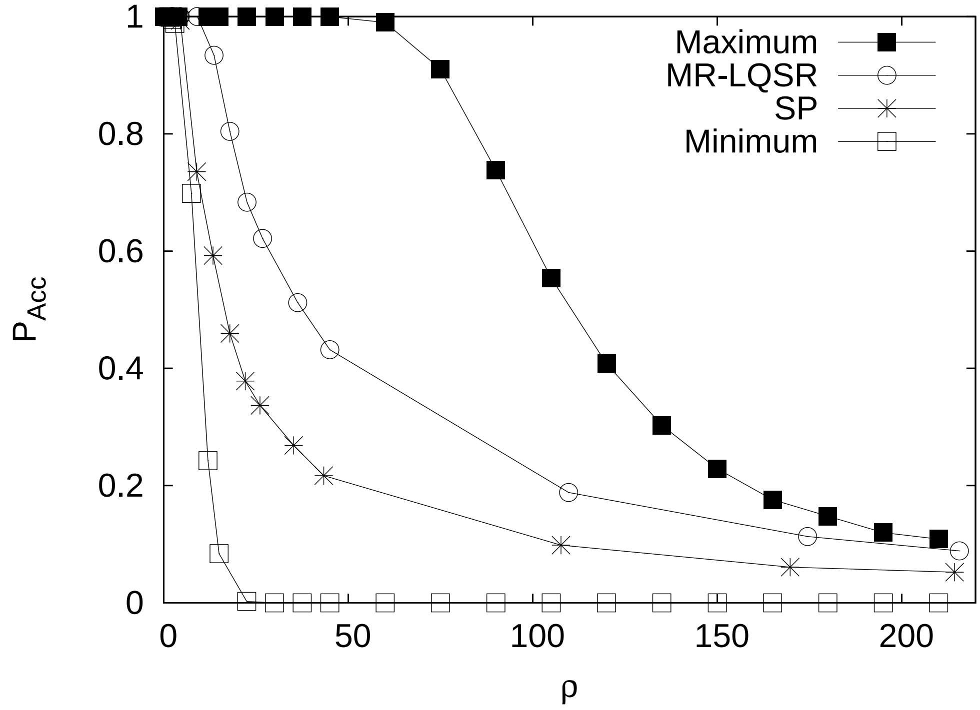

Figure 2.14: P_{Acc} vs ρ for 3-hop calls with $\mathcal{K} = 2$ and $C = 3$.

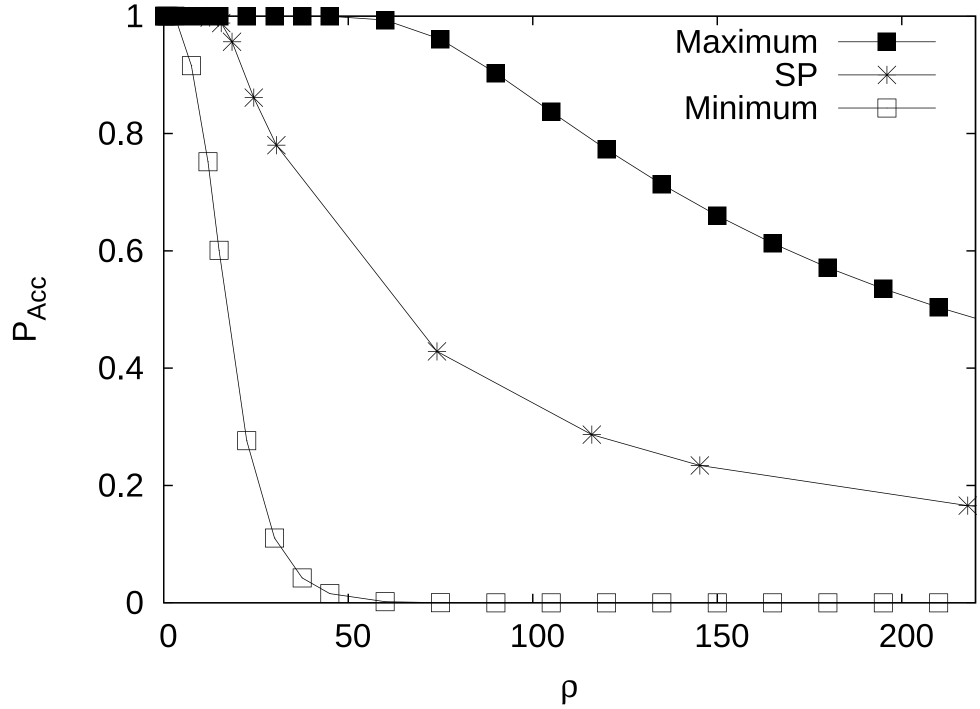

Figure 2.15: P_{Acc} vs ρ for 1-hop calls with $\mathcal{K} = 2$ and $C = 4$.

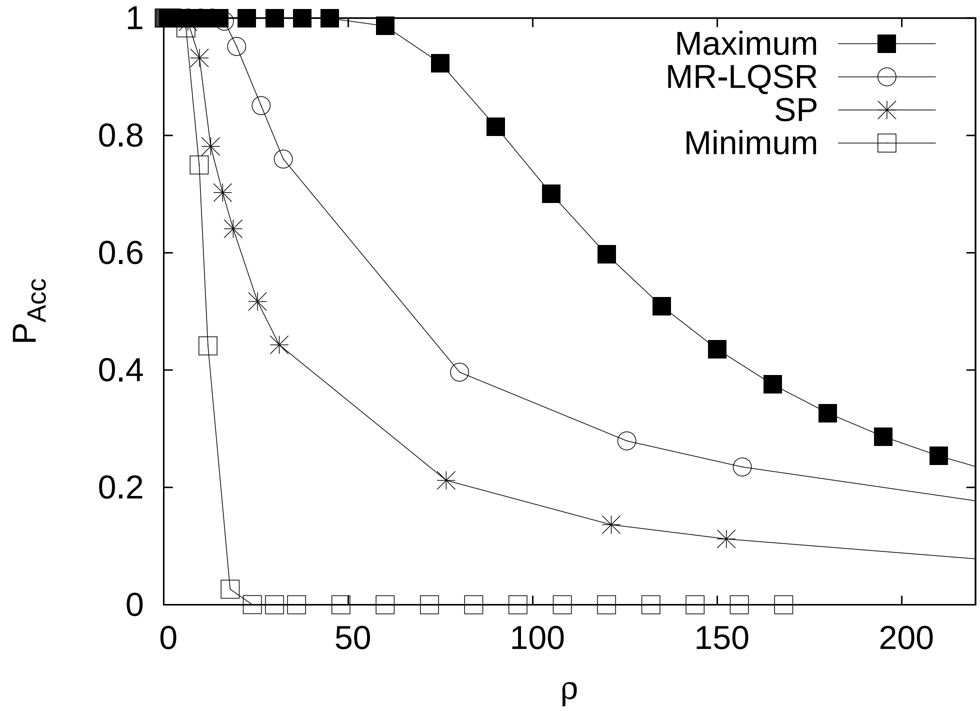

Figure 2.16: P_{Acc} vs ρ for 2-hop calls with $\mathcal{K} = 2$ and $C = 4$.

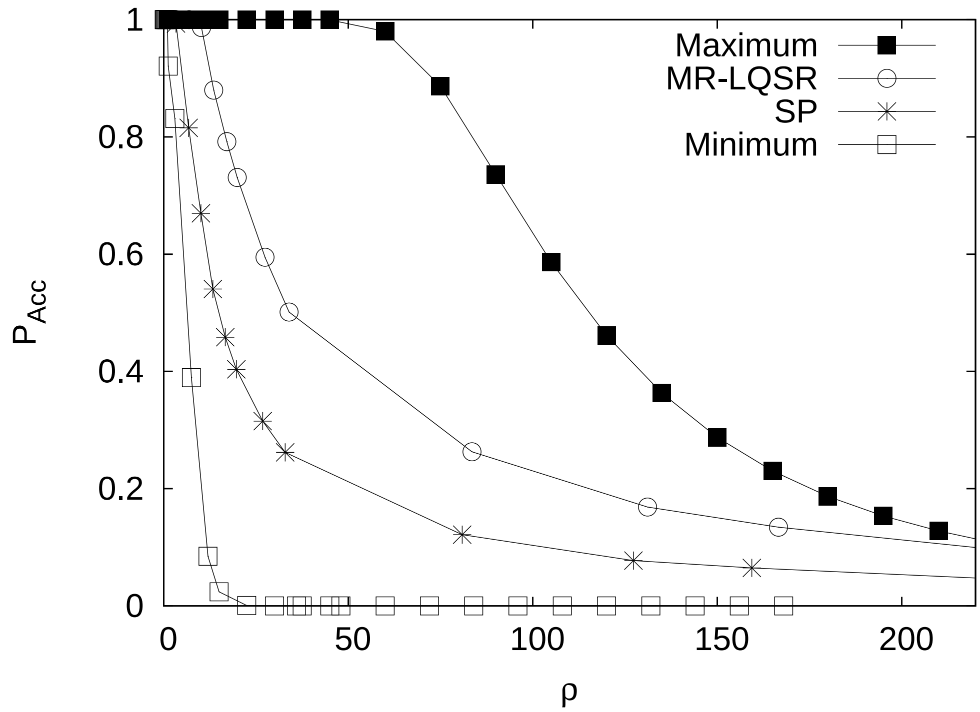

Figure 2.17: P_{Acc} vs ρ for 3-hop calls with $\mathcal{K} = 2$ and $C = 4$.

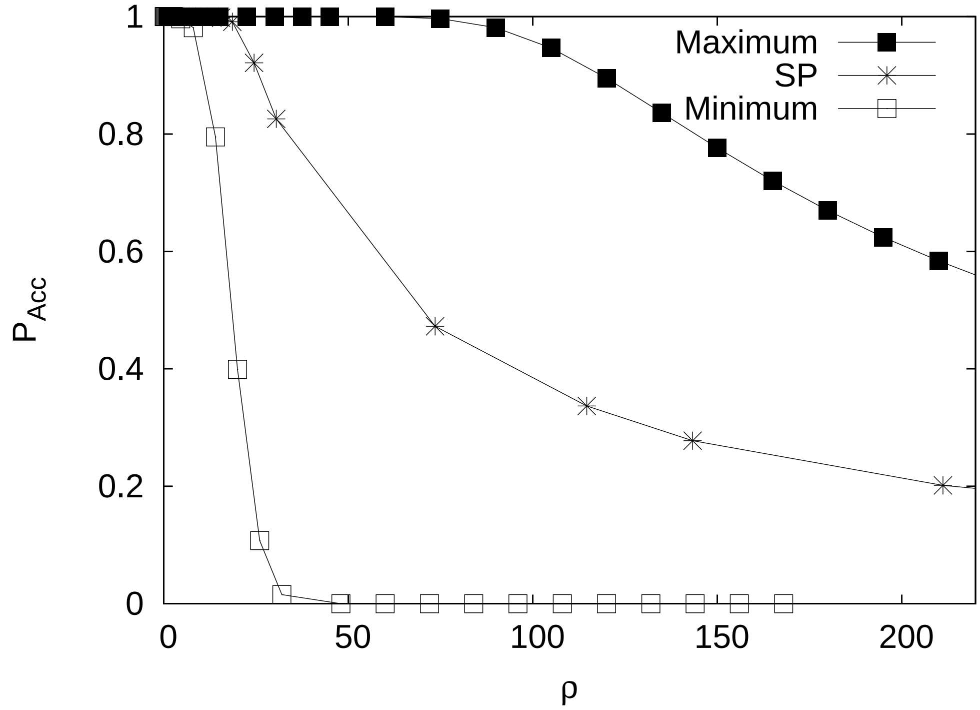

Figure 2.18: P_{Acc} vs ρ for 1-hop calls with $\mathcal{K} = 3$ and $C = 4$.

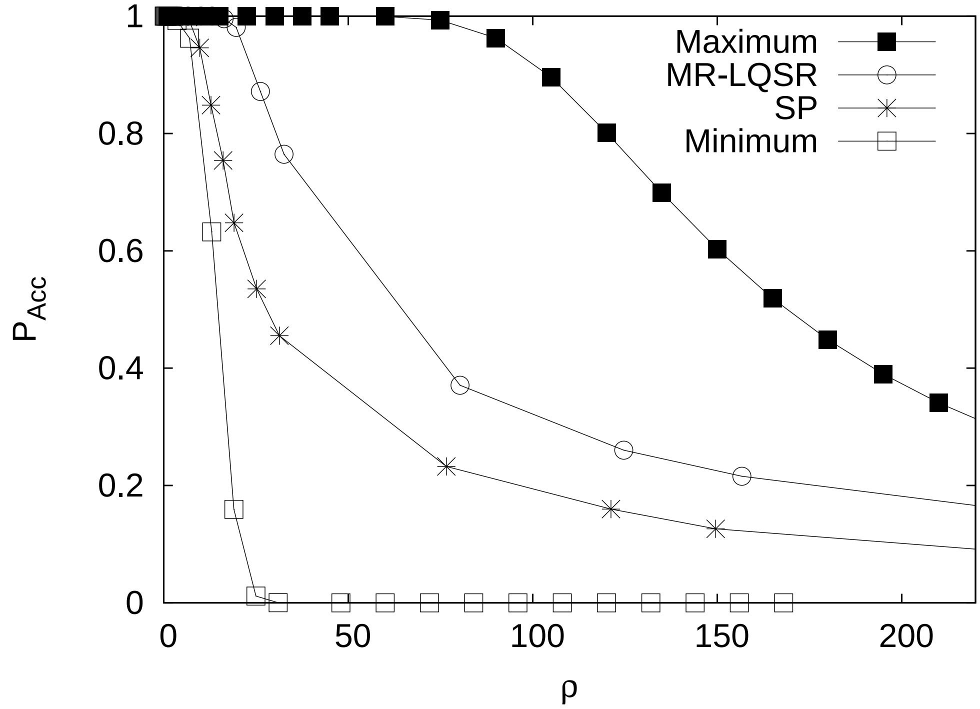

Figure 2.19: P_{Acc} vs ρ for 2-hop calls with $\mathcal{K} = 3$ and $C = 4$.

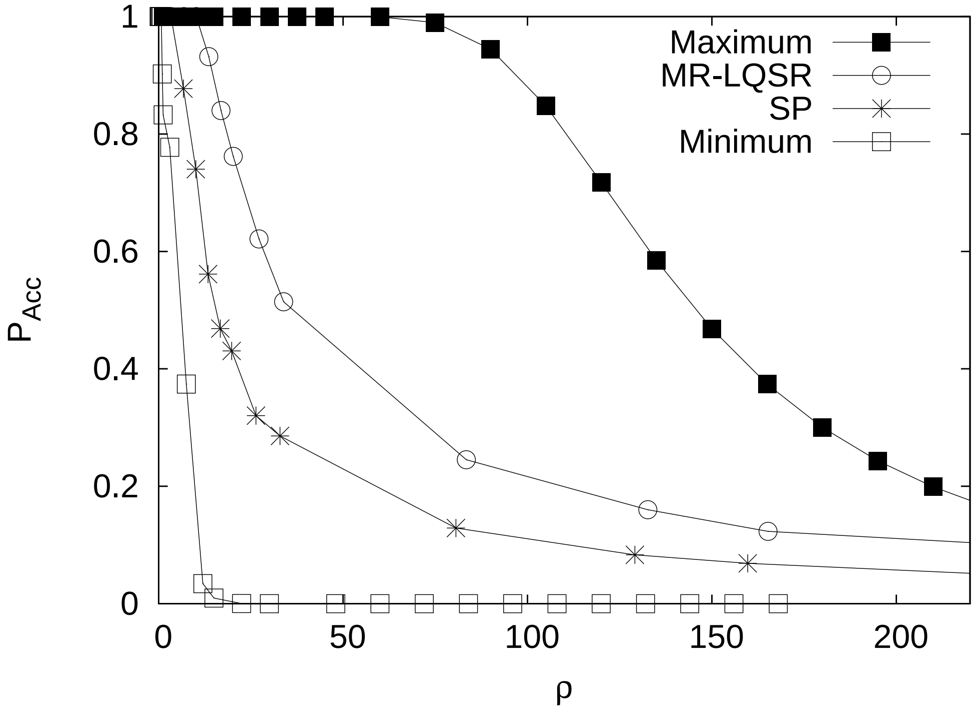

Figure 2.20: P_{Acc} vs ρ for 3-hop calls with $\mathcal{K} = 3$ and $C = 4$.

diversified assignment of channels is handled by the channel assignment algorithm). This also causes a reduction in the inter node interference in each region $R(j)$.

In all the results, we see that the probability of call acceptance lies within the theoretical bounds P_{Acc}^{Max} and P_{Acc}^{Min}. The probability of call acceptance decreases with increasing loads as expected. We observe that MR-LQSR out-performs SP routing for all multi-hop calls, under all combinations of channels and radios considered. The WCETT metric does not just consider the latency of the path, but also takes into consideration the bandwidth of the path. It chooses paths that reduce path based interference by penalizing paths that do not have a diversity of channels while giving due consideration to the length of the path. SP routing does not consider the channels assigned along a path. Thus, in dense mesh networks, the increased interference caused by not considering path based interference leads to a drop in probability of call acceptance.

The increase in probability of call acceptance with MR-LQSR emphasizes the need to consider better routing metrics in an MC-MR network. In spite of using a better routing metric, the difference between the theoretical bound and experimental values reflects the inadequacy of the metric, though the difference is partly because of our assumptions.

2.7 Summary

In this work, we have theoretically analyzed the end-to-end call acceptance in a TDMA based MC-MR WMN. We have also considered providing bandwidth guarantees in the presence of resource contention and analyzed the network performance under different routing protocols with a distributed channel assignment scheme [16]. We also studied the effect of the number of radios in each node and the number of channels available on end-to-end call acceptance via analysis and simulations. From the theoretical bounds, we observe that if the channel assignment algorithm assigns equal number of edges for all channels, in each interference region, then the end-to-end call acceptance is maximum. The routing protocol that uses the WCETT metric (MR-LQSR) out performs SP in terms of the probability of call acceptance. This clearly indicates the importance of considering path based interference in the routing protocol.

CHAPTER 3

End-to-End Flow Allocation and Channel Assignment in Multi-Channel Multi-Radio Wireless Mesh Networks with Partially Overlapped Channels

3.1 Introduction

Multi-Channel Multi-Radio Wireless Mesh Networks (MC-MR WMNs) [1] are characterized by an ad hoc backhaul, mainly consisting of static wireless mesh routers. Mesh routers have multiple radio transceivers, which allow them to communicate simultaneously with more than one neighboring mesh router, using different channels of operation available in the network, in an interference free manner i.e., a mesh router can be transmitting or receiving in a channel A with mesh router x, while simultaneously transmitting or receiving in channel B with mesh router y ($A \neq B$). The mesh backbone, which is a multihop wireless network, is the infrastructure part of the network to which the clients connect. Each mesh router acts as an access point for wireless mobile clients. The clients are unaware of the backhaul structure and directly connect to one of the wireless mesh routers.

WMNs differ significantly from other multihop wireless networks, like mobile ad-hoc networks and the power constrained wireless sensor networks, in that the mesh networks do not have mobility support and power efficiency as their major issues, as the mesh routers are usually stationary, with power not a constraint. A very important consideration for the widespread deployment of such broadband wireless networks is the support of bandwidth intensive applications. Hence, the most important design consideration for broadband WMNs, is increasing network throughput. There have been several implementation studies on WMNs to provide last mile access [24, 25] and some commercial deployments of WMNs exist which are operational [26, 27, 28].

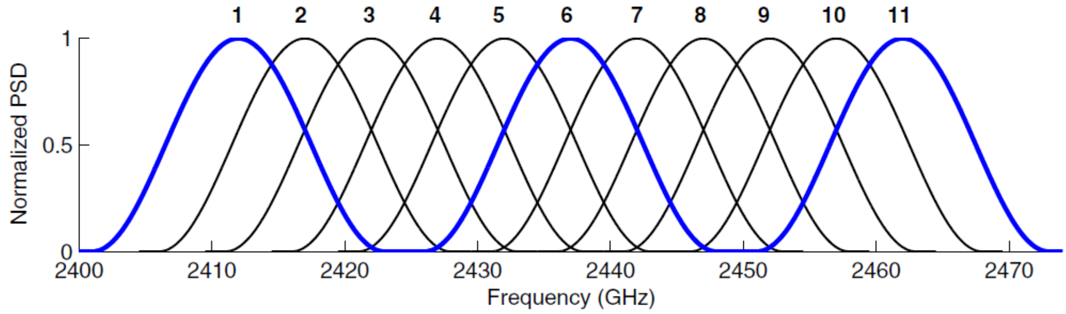

Figure 3.1: Available 11 partially overlapped channels in IEEE 802.11b/g networks. Channels 1, 6 and 11 are non-overlapping.

A major factor which is responsible for the high throughput of WMNs is the usage of multiple interfaces in each mesh router, along with the availability of multiple channels for transmission in the network. In [14], the authors show that there is a non-linear increase in system capacity with the increase in the number of radios at each mesh node. However, due to the limited number of channels available, channel contention is possible, leading to collisions and interference. Thus, in order to effectively utilise the multiple radios and multiple channels, efficient channel assignment algorithms are needed.

WMNs generally have a lower network throughput compared to wired networks, because, due to the existence of interference, simultaneous transmissions are prohibited. Thus, reducing the effect of interference is a very important consideration in WMNs. A typical wireless signal spans over several frequencies, but typically has its energy concentrated only over a narrow range of frequencies. The available frequencies for transmission are heavily regulated by regulatory bodies. This is done in order to cater to various wireless technologies, that use several different kinds of signal modulation and access mechanisms, that may not be compatible with each other. The IEEE 802.11b/g standards, provide 11 channels for transmission in the 2412-2462 MHz range [3], of which only 3 channels are non-overlapping channels (Figure 3.1). Recent studies have shown that the system throughput can further be increased when both non-overlapped channels and partially overlapped channels are used [29, 30].

An overwhelming number of recent as well as classical papers, have studied the channel assignment problem, in association with several other related problems such as routing and congestion control, in great detail [4, 14, 15, 16, 18, 31, 32, 33]. However, all these works only consider non-overlapping channels in their analysis. Our contributions in this work are as follows:

- We jointly study the problem of channel assignment and flow allocation, assuming both non-overlapped and partially overlapped channels are used in the network, such that the aggregate network throughput is maximized, and the average queueing delay in the network is minimized. We consider maximizing the useful aggregate end-to-end communication throughput, rather than sum of link capacities.
- We propose a Mixed Integer Linear Program (MILP) formulation and study the performance of the problem under different objectives. The linear nature of the problem along with a low number of binary variables decreases the computation complexity of the problem.
- Apart from the standard objective of maximizing network throughput, we show the importance of considering other important factors, such as the average queueing delay in the network in order to increase the resilience of the network.
- Given the topology of the WMN, we propose a heuristic algorithm for the channel assignment and flow allocation problem in order to scale the solution to bigger networks.

3.2 Related Work

MC-MR multi-hop wireless networks have recently received extensive research attention due to their potential future applications. A lot of work has been done in trying to exploit the multiple channels and multiple radios to increase the network capacity. Draves et al. [4], present a new routing metric Weighted Cumulative Expected Transmission Time (WCETT), and a corresponding Multi-Radio Link Quality Source Routing (MR-LQSR) protocol to find a high throughput path between a source node and a destination node in multi-radio, multi-hop wireless networks. In [14], the authors have proposed one of the first 802.11 based multi-channel multi-hop WMN architecture. They have developed a set of centralized algorithms for channel assignment, bandwidth allocation, and routing. They show that the capacity of the network increases non-linearly with an increase in the number of radios. In a later paper [15] they also present distributed channel assignment and routing algorithms utilizing only local traffic load information. Chen et al. [34] propose an MILP formulation for channel assignment and routing which they solve using simulated annealing. In [31], the authors propose a constant bound approximation algorithm to solve the joint channel assignment, routing, and scheduling problem for fair rate allocation. In [35], the authors study a similar problem and derive upper bounds for the achievable throughput. They also propose two channel assignment heuristics: a static link channel assignment and a dynamic link channel assignment.

In [32], the authors develop a column generation based approach to solve the joint routing and scheduling problem. In [36], Tang et al. proposed interference aware topology control and routing schemes for QoS provisioning. Later in [8], they presented polynomial time optimal schemes to compute maximum throughput and fair bandwidth allocation. In [16], a distributed channel assignment algorithm which uses a skeleton assisted channel allocation strategy is proposed. It relies on the construction of a spanning subgraph (called skeleton) of the connectivity graph using a distributed algorithm LMST (Local Minimum Spanning Tree) [17]. The proposed Safe protocol uses the constructed skeleton to assign channels while preserving connectivity. In [18], the authors consider a fixed MC-MR WMN and propose a MILP based static channel assignment scheme that maximizes the number of bidirectional links that can be activated simultaneously, subject to interference constraints.

In [29] and [30], the authors show that the network capacity can be improved when both non-overlapped and partially overlapped channels are being used. They introduce the idea of the channel overlap factor for quantifying the overlap between two channels. The authors construct simple analytical and empirical models for interference that occurs in IEEE 802.11 networks and illustrate several schemes to incorporate partial overlapping channels in wireless ad hoc networks and WMNs. The capacity of wireless ad hoc networks is increased by improving spatial channel reuse, and that of WMNs by allowing nodes with single interface to communicate more efficiently with its neighbors in 802.11 ad hoc mode. In [37], a channel assignment and link scheduling algorithm, based on a Genetic Algorithm is presented. The authors study the effect of several factors such as network topology, node density, and node distribution, on the network performance. A Joint Optimal Channel Assignment and Congestion Control algorithm (JOCAC) which is formulated as a non-linear problem, is presented in [38]. The JOCAC is presented as a decentralized utility maximization problem with constraints that arise from the interference of the neighboring transmissions. In [39], the authors relax the binary interference model and introduce the notion of *partial interference*. The authors present an analytical framework to characterize *partial interference* in a single-channel wireless network under unsaturated traffic conditions, and use 802.11b with basic access scheme and differential binary phase shift keying as an illustration.

In [40], the authors study rate allocation for a set of end-to-end communication sessions in MC-MR WMNs. A cross layer scheme to solve the joint rate allocation, routing, scheduling, power control, and channel assignment problems is provided. Fairness is addressed using both a simplified max-min fairness model and the well-known proportional fairness model. In [33], Wang and Kar propose cross layer primal and dual based algorithms to compute proportional fair end-to-end rate allocation in a multihop Aloha-based wireless network.

3.3 System Model

3.3.1 Network Model

We consider a WMN with N mesh nodes randomly dispersed in a region of area A. We use the term mesh node and node interchangeably to refer to the stationary mesh routers that constitute the WMN backbone. There are $\mathcal{K}$ radios ($\mathcal{K} \geq 2$) in each node and C channels ($C \geq \mathcal{K}$) are available for transmission in the network. We use omnidirectional antennas, for communication at the physical layer. We assume that the network topology has been pre-determined. We represent the network in the form of an undirected graph $G(\mathcal{N}, \mathcal{E})$ where $\mathcal{N}$ represents the set of nodes in the network ($N = |\mathcal{N}|$), and $\mathcal{E}$, the set of all logical links ($E = |\mathcal{E}|$). A logical link from node A to node B is represented as $(a, b) \in \mathcal{E}$. We assume the link connectivity to be symmetric, i.e., link $(a, b) \in \mathcal{E}$ *iff* link $(b, a) \in \mathcal{E}$. A link (a, b) exists *iff* nodes a and b are in the transmission range (TR) of each other. The symmetric link connectivity assumption forces the transmission range at each node to be the same. We also assume that as long as nodes a and b are in the transmission range of each other, there is no degradation of quality of the link (a, b).

Each node a transmits with a power P_a. This transmission power is same across all channels. There is a constraint on the maximum power level that a node can use for transmission, which is identical for all nodes. We denote this maximum power level by P^{Max}. We assume half-duplex operation at each radio to prevent self-interference and only consider unicast communication.

3.3.2 Network Traffic Model

We assume that a network traffic profiler exists, which provides information regarding the traffic demands of the end-to-end communication sessions that exist in the network. Each communication session is uniquely identified by a triple (i, j, t_{ij}), where i

is the source, j is the destination, and t_{ij} is the traffic demand for the communication session. The set of end-to-end communication sessions that exist in the network, is denoted by $\mathcal{T}$, i.e., $\mathcal{T} = \{(i, j, t_{ij})|i \in \mathcal{N}, j \in \mathcal{N},\ t_{ij} > 0\}$. We assume that the routing algorithm provides us with multiple paths from a source to a destination. For example, we could consider maintaining information about the successive shortest paths [41], between the source and destination of a communication session. These alternate paths between a node i and node j, for an end-to-end communication session (i, j, t_{ij}), are represented by an indicator variable $\mathcal{R}^e_{ij,k}$, $e \in \mathcal{E}$, where k denotes the k^{th} path between node i and node j. $\mathcal{R}^e_{ij,k}$ is 1 if the k^{th} path from node i to node j as given by the routing protocol, passes through link e. We use the term flow and communication session interchangeably. The total achieved throughput for a flow (i, j, t_{ij}) is denoted by x_{ij}. We assume that the traffic for a flow is *infinitely divisible*. We make this assumption, in order to support multi-path routing for each flow. We denote the achieved throughput on the k^{th} path between i and j as $x_{ij,k}$. We have, $x_{ij} = \sum_k x_{ij,k}$.

We note that for any flow (i, j, t_{ij}), a value of the achieved throughput (x_{ij}) that is greater than the throughput demand (t_{ij}) is not useful.

3.3.3 Model for Partially Overlapped Channels

We model the partial overlap between channels as in [30]. Let T be a transmitter operating on frequency F_T, and correspondingly let R be the receiver tuned to a frequency F_R. We denote the transmitter's signal power distribution across the frequency spectrum by $S_T(f)$, and the frequency response of the receiver's band pass filter by $B_R(f)$. In order to model the overlapping among different channels, we consider the *interference factor* $I(f)$, which is defined as

$$I(\tau) = \int_{-\infty}^{+\infty} S_T(f) B_R(f - \tau)\, df \qquad (3.1)$$

The interference factor, as defined above, captures the overlap between transmission and reception on two different frequencies separated by τ ($\tau = F_T - F_R$ in the above equation). The above equation applies to any band limited signal, irrespective of the modulation scheme used (Direct-Sequence Spread Spectrum (DSSS), Orthogonal Frequency Division Multiplexing (OFDM) etc.).

Considering the IEEE 802.11 standard using DSSS modulation, we have the transmit spectrum mask (which specifies the upper limit for the power permissible for each frequency of the transmitted channel) as shown in Figure 3.2. In the figure, F_c de-

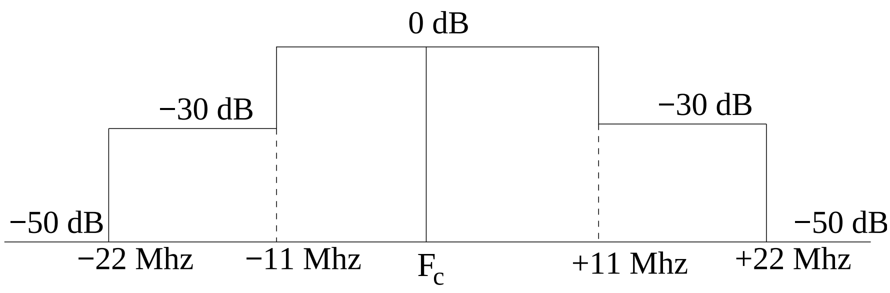

Figure 3.2: Transmit spectrum mask for IEEE 802.11 DSSS modulation.

notes the center frequency of channel c. At F_c, the transmit spectrum mask limits relative output power to $0\,dB$ (relative power $= 10log(\frac{P_{out}}{P_{in}})$), i.e., the output power is equal to the input power and the signal is passed unaffected. At frequencies beyond $F_c + 11$ MHz and $F_c - 11$ MHz, the power is attenuated down by $-30dB$ and further to $-50dB$ at $F_c \pm 22$ MHz. We assume such a mask for our network, and assume that the transmitter's signal power distribution is exactly that of the transmit spectrum mask (Figure 3.2). We also assume that the receiver's band pass filter is same as the transmit spectrum mask. These assumptions can be justified, by noting that, the use of the same filter for transmitting a signal and band limiting the received signal in a transceiver is advantageous. Thus, we have:

$$B_R(f) = S_T(f) = \begin{cases} -50\,dB & |f - F_c| > 22\,MHz \\ -30\,dB & 11\,MHz < |f - F_c| < 22\,MHz \\ 0\,dB & |f - F_c| < 11\,MHz \end{cases} \tag{3.2}$$

Using the above assumptions, we define a $C \times C$ symmetric *channel overlapping matrix* $\mathcal{O}$ ($\mathcal{O} = [\mathbf{o}]_{\mathbf{ij}}$) such that

$$[o]_{ij} = \begin{cases} 1 & i = j \\ \frac{I(5|i-j|)}{I(0)} & i \neq j \end{cases} \tag{3.3}$$

where $[o]_{ij}$ represents the entry in the i^{th} row and j^{th} column of the matrix $\mathcal{O}$. In the above equation, we have replaced τ by $5|i - j|$. This follows from the fact that, in the IEEE 802.11 standard, the channels are separated by a gap of 5 MHz. Figure 3.3 shows the variation of the channel overlap factor ($[o]_{ij}$) with the channel separation. We note that, as the channel separation increases, the overlap factor decreases, until it reaches 0 for a channel separation ≥ 5.

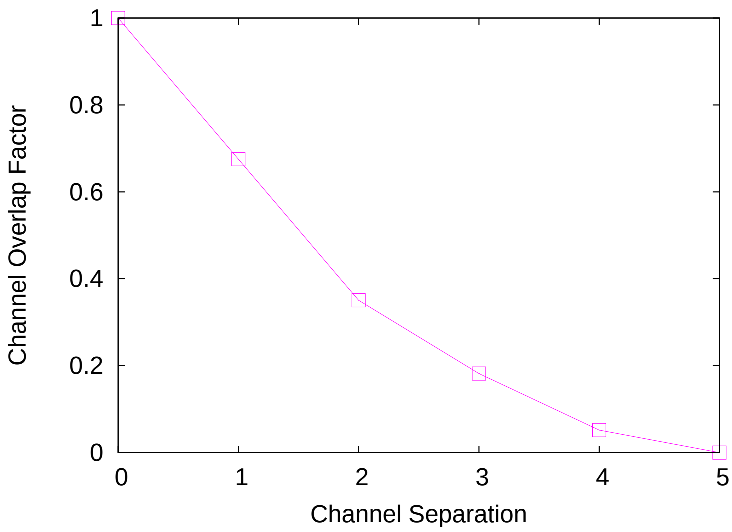

Figure 3.3: Variation of channel overlap factor with channel separation.

3.3.4 Interference Model

We use the physical model proposed in [9] to model the impact of interference. Consider two links (a, b), $(c, d) \in \mathcal{E}$, which are assigned channel i and j respectively. If the interference power of link (a, b) causes the *signal to interference plus noise ratio* on link (c, d) to fall below a threshold ($SINR_{Min}$), then the transmitter of link (a, b) is within the interference range of the receiver of the link (c, d). Let G_{ad} denote the channel gain for the nodes a and d, which depends on path-loss, channel fading, and shadowing. The interference power from link (a, b) on link (c, d) can be estimated as:

$$\text{Interference power from link (a,b) on link (c,d)} = [o]_{ij}\, G_{ad}\, P_a \tag{3.4}$$

Using the above Equation (3.4), we have:

$$\frac{G_{cd}\, P_c}{N_0 + [o]_{ij}\, G_{ad}\, P_a} < SINR_{Min} \tag{3.5}$$

if the transmitter of link (a, b) interferes with the receiver of link (c, d). N_0 refers to the thermal noise power at the receiver of link (c, d). The $SINR_{Min}$ value is generally determined using some QoS requirement, such as Bit Error Rate (BER).

We can model the path loss G_{cd} and G_{ad} using the Friis Space Model [42]:

$$G_{ad} = \frac{\alpha}{(r_{ad})^k} \tag{3.6}$$

where r_{ad} is the Euclidean distance between nodes a and d, k is a path-loss exponent, and α is a constant that depends on the transmitter and receiver antenna gains and the signal wavelength. Using (3.5) and (3.6) we have the following, if link (a, b) interferes with link (c, d):

$$r_{ad} < \sqrt[k]{\left(\frac{\alpha\, P_a}{\frac{G_{cd}\, P_c}{SINR_{Min}} - N_0}\right)[o]_{ij}} \tag{3.7}$$

From Equation (3.7), we note that the interference range varies with the channel assigned to the links. The lesser the overlap between the channels, shorter is the interference range. We also note that the interference range depends only on the channel separation $|i-j|$ as shown in Figure 3.4. Figure 3.4 shows the interference ranges for varying channel separations, where i and j are the channels assigned to links (a, b) and (c, d) respectively. From the figure, we see that the interference range for the case of $|i-j| = 0$ is the highest and that the interference range decreases with increasing $|i-j|$ values. In the figure, links (a, b) and (c, d) interfere with each other, if $|i-j| = 0$ *or* 1 *or* 2.

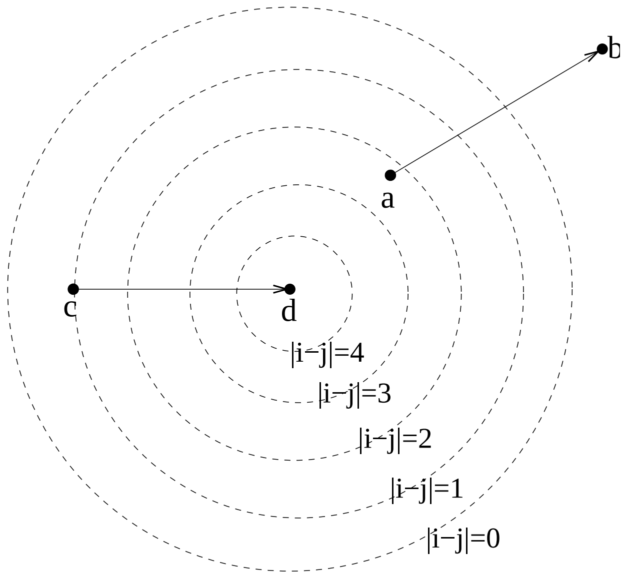

Figure 3.4: Variation of interference range with channel separation.

Representing the function on the RHS of Equation (3.7) by $\mathcal{F}(a, c, d, i, j)$ we can rewrite Equation (3.7) as:

$$r_{ad} < \mathcal{F}(a, c, d, i, j) \tag{3.8}$$

We note that, two transmissions with a common receiver are not allowed to be made simultaneously from the above definitions, considering all possible cases of the orientations of links (a, b) and (c, d) as in Figure 3.5, we have the following necessary and sufficient condition, for two links (a, b) and (c, d) to interfere with each other:

$$r_{cb} < \mathcal{F}(c, a, b, i, j) \quad or \quad r_{ad} < \mathcal{F}(a, c, d, i, j) \tag{3.9}$$

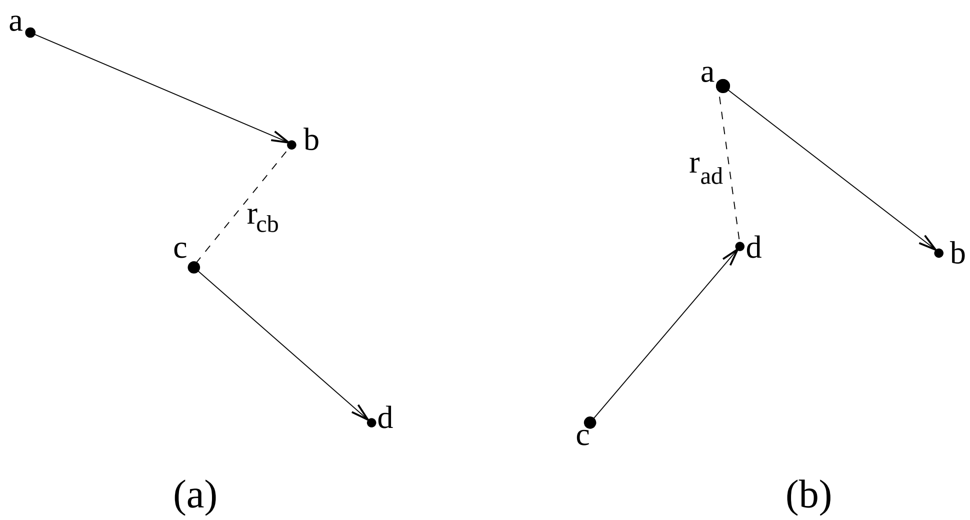

Figure 3.5: Cases to consider to decide if two links interfere. (a) shows the case where transmitter c interferes with the transmission over the link (a, b). (b) shows the case where transmitter a causes interference with the transmission over the link (c, d).

In order to capture the above interference relations between the links of the network, we define the *Link Interference Matrix* (**LIM**). We note that, in our network, the LIM is not just determined by the location of the links in the logical topology, as in [18], but also by the channels assigned to the links. Consider two links $e = (a, b)$ and $e' = (c, d)$. The Link Interference Matrix $\mathbf{LIM_{e,e'}}$ for these two links is a symmetric $C \times C$ matrix, with the entry in the i^{th} row and j^{th} column being 1 if link e interferes with link e'. These interference relations can be determined using Equation (3.9). Thus, we have:

$$[\mathbf{LIM_{e,e'}}]_{\mathbf{ij}} = \begin{cases} 1 & if\, r_{cb} < \mathcal{F}(c, a, b, i, j) \quad or \quad r_{ad} < \mathcal{F}(a, c, d, i, j) \\ 0 & otherwise \end{cases} \tag{3.10}$$

3.4 ILP Formulation

3.4.1 Channel Assignment Model

We consider the static channel assignment problem, in a network with N nodes, E links, with each node having $\mathcal{K}$ radios, and a total of C non-overlapped and partially overlapped channels available in the network ($C = 11$ for IEEE 802.11b networks). Given an assignment of channels to radios, a link (a, b) exists if and only if they share a common channel. In order to maintain network connectivity, we ensure that the channel assignment algorithm is such that, it assigns a channel to every logical link in the network.

For a link $e = (a, b) \in \mathcal{E}$, we define a $C \times 1$ zero-one *channel assignment vector* $\bar{\mathbf{w}}_{\mathbf{e}}$, where the i^{th} element of $\bar{\mathbf{w}}_{\mathbf{e}}$ is 1, if link e is assigned channel i, otherwise it is zero. An $m \times n$ matrix $\mathbf{M}$ is zero-one if each entry $\in$ {0,1}. The definition of a zero-one vector follows. We also define a $C \times 1$ zero-one vector $\bar{\mathbf{y}}_{\mathbf{n}}$ for every node $n \in \mathcal{N}$, such that the i^{th} element of $\bar{\mathbf{y}}_{\mathbf{n}}$ is 1, if node n has a radio tuned to channel i, otherwise it is zero.

We now define some constraints, which decide the feasibility of channel assignment. In order to maintain the connectivity requirement, every link $e \in \mathcal{E}$ must be assigned a channel. We assume that only one channel is assigned to each link e. Thus, we must have:

$$\sum_{i=1}^{C} [\mathbf{w_e}]_{\mathbf{i}} = \mathbf{1} \qquad \forall\, \mathbf{e} \in \mathcal{E} \tag{3.11}$$

If one of the incident links e, at a node $n \in \mathcal{N}$ is assigned a channel c, i.e., $[\mathbf{w_e}]_{\mathbf{c}} = \mathbf{1}$, then one of the radios of node n must be tuned to channel c, i.e., $[\mathbf{y_n}]_{\mathbf{c}} = \mathbf{1}$. Restating the condition, if a link $e = (a, b)$ is assigned channel c, then both the incident nodes of that link (a and b), must have a radio tuned to channel c. Thus, we have:

$$[\mathbf{y_n}]_{\mathbf{c}} \geq [\mathbf{w_e}]_{\mathbf{c}} \qquad \forall\, \mathbf{e} \in \mathcal{E}, \mathbf{n} \in \mathbf{Inc}(\mathbf{e}), \mathbf{1} \leq \mathbf{c} \leq \mathbf{C} \tag{3.12}$$

where $Inc(e)$ denotes the incident nodes of link e. From the previous constraint, we note that $[\mathbf{y_n}]_{\mathbf{c}}$ is free to take on a value of 1, even if none of the incident links on node n are assigned channel c. However, the next constraint (3.13), which is an upper bound on the maximum number of channels that can be assigned to a node n, ensures that feasibility is not violated due to spurious allocations. The constraint on the maximum number of distinct channels that can be assigned to edges incident on a node n is given by:

$$\sum_{i=1}^{C} [\mathbf{y_n}]_{\mathbf{i}} \leq \mathcal{K} \qquad \forall\, \mathbf{n} \in \mathcal{N} \tag{3.13}$$

Finally, using the definition of $\mathbf{LIM_{e,e'}}$, we have the following condition for two links $e = (a, b)$ and $e' = (c, d)$ to interfere with each other:

$$[\mathbf{w_e}]_\mathbf{i}[\mathbf{LIM_{e,e'}}]_\mathbf{ij}[\mathbf{w_{e'}}]_\mathbf{j} = \begin{cases} 1 & \text{If Links } e \text{ and } e' \text{ interfere} \\ 0 & \text{otherwise} \end{cases} \tag{3.14}$$

where links e and e', are assigned channels i and j respectively.

3.4.2 Flow Rate at each Link

Let $c(e)$ denote the achievable bandwidth on link e. $\alpha_{e,c}$ $(0 \leq \alpha_{e,c} \leq 1)$ denotes the normalized flow rate of link e on channel c (the fraction of time, a link e, which is assigned channel c, is active). The achieved throughput of link e, if it is assigned channel c, denoted as $f_{e,c}$, is related to $c(e)$ and $\alpha_{e,c}$ as:

$$f_{e,c} = \alpha_{e,c} \times c(e) \qquad \forall\, e \in \mathcal{E}, 1 \leq c \leq C \tag{3.15}$$

From [31], a sufficient condition for interference free link scheduling is:

$$\frac{f_{e,i}}{c(e)} + \sum_{e' \in \mathcal{E},\, e' \neq e} [\mathbf{w_e}]_\mathbf{i}[\mathbf{LIM_{e,e'}}]_\mathbf{ij}[\mathbf{w_{e'}}]_\mathbf{j} \times \frac{\mathbf{f_{e',j}}}{\mathbf{c(e')}} \leq \mathbf{1} \qquad \forall\, \mathbf{e} \in \mathcal{E}, \mathbf{1} \leq \mathbf{i}, \mathbf{j} \leq \mathbf{C} \tag{3.16}$$

If two logical links e and e' interfere with each other, they cannot be active simultaneously. Equation (3.16) enforces this condition in the network. Assuming all links in the network have a nominal data transmission rate of c_0, Equation (3.16) can be written as:

$$f_{e,i} + \sum_{e' \in \mathcal{E},\, e' \neq e} [\mathbf{w_e}]_\mathbf{i}[\mathbf{LIM_{e,e'}}]_\mathbf{ij}[\mathbf{w_{e'}}]_\mathbf{j} \times \mathbf{f_{e',j}} \leq \mathbf{c_0} \qquad \forall\, \mathbf{e} \in \mathcal{E}, \mathbf{1} \leq \mathbf{i}, \mathbf{j} \leq \mathbf{C} \tag{3.17}$$

However, we note that, the channels assigned to the links is not known in advance. So, we can rewrite Equation (3.17) as:

$$f_e + \sum_{e' \in \mathcal{E},\, e' \neq e} \left(\sum_{i=1}^{C} \sum_{j=1}^{C} [\mathbf{w_e}]_\mathbf{i}[\mathbf{LIM_{e,e'}}]_\mathbf{ij}[\mathbf{w_{e'}}]_\mathbf{j} \right) \times f_{e'} \leq c_0 \qquad \forall\, e \in \mathcal{E} \tag{3.18}$$

where f_e is the achieved throughput of link e, after channel assignment. We note that Equations (3.18) and (3.17) are equivalent.

We now define constraints for the end-to-end communication sessions in the network. Using our earlier defined notion of useful throughput (Section 3.3.2), we add a constraint that ensures that x_{ij} actually refers to the useful throughput:

$$x_{ij} \leq t_{ij} \qquad \forall \text{ flows } (i, j, t_{ij}) \in \mathcal{T} \tag{3.19}$$

In order to ensure fairness among flows and prevent starvation of flows, we strive to guarantee a fraction of the actual demanded flow. We introduce a parameter λ which represents the minimum guaranteed flow fraction of the actual demanded flow. Thus, we must have:

$$x_{ij} \geq \lambda t_{ij} \qquad \forall \text{ flows } (i, j, t_{ij}) \in \mathcal{T} \tag{3.20}$$

Note that λ affects the feasibility of the channel assignment and flow allocation. A very high value for λ may lead to infeasibility of the final MILP.

We now rewrite Equation (3.18) in terms of the allocated flow values that pass through a given link e. We have 3 cases to consider:

3.4.2.1 Fixed Path Routing

In this type of routing, we use a single fixed path for each flow (e.g., the shortest path) which is decided before the channel assignment is performed. For this case, the load on a link e, assuming link e is assigned channel c (equivalent to $f_{e,c}$), as determined by the routing algorithm and the demand of the flows, is given by:

$$f_{e,c} = \sum_{(i,j,t_{ij}) \in \mathcal{T}} \mathcal{R}^{e}_{ij,1} \times x_{ij,1} \tag{3.21}$$

3.4.2.2 Multi Path Routing

In this type of routing, we use multiple paths provided by the routing algorithm, for each flow. For this case, the load on a link e assuming link e is assigned channel c (equivalent to $f_{e,c}$) is given by:

$$f_{e,c} = \sum_{(i,j,t_{ij}) \in \mathcal{T}} \sum_{k} \mathcal{R}^{e}_{ij,k} \times x_{ij,k} \tag{3.22}$$

3.4.2.3 Single Path Routing

In this type of routing, we choose a single path among the many available paths for each flow. However, the choice of the path for each flow is done jointly with the channel assignment. For a flow $(i, j, t_{ij}) \in \mathcal{T}$, let $r_{ij,k}$ be an indicator variable, such that $r_{ij,k}$ is 1 if the k^{th} path between node i and node j is chosen for routing. Since we finally need a single path for each flow, we must have:

$$\sum_{k} r_{ij,k} = 1 \qquad \forall \text{ flows } (i, j, t_{ij}) \in \mathcal{T} \tag{3.23}$$

We note that for this case, the relation between x_{ij} and $x_{ij,k}$ is modified as:

$$x_{ij} = \sum_k r_{ij,k} \times x_{ij,k} \tag{3.24}$$

The load on a link e, assuming link e is assigned channel c (equivalent to $f_{e,c}$) is given by:

$$f_{e,c} = \sum_{(i,j,t_{ij}) \in \mathcal{T}} \sum_k r_{ij,k} \times \mathcal{R}^e_{ij,k} \times x_{ij,k} \tag{3.25}$$

3.4.3 Objective Function

We consider the fixed path routing scheme in our formulation. In this case, given the set of end-to-end communication sessions $\mathcal{T}$, and the unique routing path for each flow, we can calculate the aggregate load on a logical link $e = (a, b)$, denoted by $l(e)$, by substituting $x_{ij,1}$ with t_{ij} in Equation (3.21). The channel assignment algorithm should ideally be able to support a major fraction of the demanded flow throughputs, and hence we would like our objective function (Obj_1) to *maximise* the aggregate end-to-end flow allocations, i.e.,

$$Obj_1 : \qquad \textbf{maximize} \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}}) \in \mathcal{T}} \mathbf{x_{ij}} \tag{3.26}$$

However, we note that based on results from queuing theory, when the link utilization is close to 1, the queuing delay at the end nodes of the link tends to be very large [43, 44]. The link utilization is defined as the ratio of the aggregate load on a link $l(e)$ to the effective data rate of the link $f_{e,c}$. A network with high queueing delays, is prone to congestion. We would thus also like to *minimise* the maximum link utilization (Obj_2) in order to reduce the chance of network congestion, while still managing the network capacity according to the expected traffic load, i.e.,

$$Obj_2 : \qquad \textbf{minimize} \left(\textbf{maximum} \, \frac{\mathbf{l(e)}}{\mathbf{f_{e,c}}} \right) \equiv \textbf{maximize} \left(\textbf{minimum} \, \frac{\mathbf{f_{e,c}}}{\mathbf{l(e)}} \right) \tag{3.27}$$

We manage these two conflicting objectives, by considering a weighted combination of the two objectives as our final objective function (Obj):

$$Obj : \qquad Maximize \left(\psi \sum_{(i,j,t_{ij}) \in \mathcal{T}} x_{ij} + (1 - \psi) \left(\textbf{minimum} \, \frac{\mathbf{f_{e,c}}}{\mathbf{l(e)}} \right) \right) \tag{3.28}$$

The weights given to either of the objectives, represents the importance of that particular objective. In highly loaded networks, the queueing delay becomes very high and hence the queueing component (Obj_2) must be given a higher weight. In lightly loaded networks, queuing delays are small and attaining the demanded throughput is more important, and hence the throughput component (Obj_1) must be given a higher weight.

Using Equations (3.11), (3.12), (3.13), (3.18), (3.19), (3.20), (3.21), and (3.28), we can summarize the Mixed Integer Non-Linear Program formulation for the channel assignment and flow allocation problem as follows:

Formulation 1 Mixed integer non-linear program formulation for the joint channel assignment and flow allocation problem.

Maximize : $\psi \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}} \mathbf{x_{ij}} + (\mathbf{1}-\psi)\mathcal{Q}$
subject to

$$\textstyle\sum_{i=1}^{C}[\mathbf{w_e}]_\mathbf{i} = \mathbf{1} \qquad \forall\, \mathbf{e} \in \mathcal{E}$$

$$\textstyle\sum_{i=1}^{C}[\mathbf{y_n}]_\mathbf{i} \leq \mathcal{K} \qquad \forall\, \mathbf{n} \in \mathcal{N}$$

$$[\mathbf{y_n}]_\mathbf{c} \geq [\mathbf{w_e}]_\mathbf{c} \qquad \forall\, \mathbf{e} \in \mathcal{E}, \mathbf{n} \in \mathbf{Inc}(\mathbf{e}), \mathbf{1} \leq \mathbf{c} \leq \mathbf{C}$$

$$\textstyle\sum \mathcal{R}^{e}_{ij,1} x_{ij,1} + \left(\sum_{e'} \left(\left(\sum_{m=1}^{C}\sum_{n=1}^{C}[\mathbf{w_e}]_\mathbf{m}[\mathbf{LIM_{e,e'}}]_\mathbf{mn}[\mathbf{w_{e'}}]_\mathbf{n}\right) \times \sum \mathcal{R}^{e'}_{ij,1} x_{ij,1}\right)\right) \leq c_0 \qquad \forall\, e, e' \in \mathcal{E}, e \neq e', (i,j,t_{ij}) \in \mathcal{T}$$

$$x_{ij} \geq \lambda t_{ij} \qquad \forall \text{ flows } (i,j,t_{ij}) \in \mathcal{T}$$

$$x_{ij} \leq t_{ij} \qquad \forall \text{ flows } (i,j,t_{ij}) \in \mathcal{T}$$

$$\mathcal{Q} \leq \frac{\sum_{(i,j,t_{ij})\in\mathcal{T}} \mathcal{R}^{e}_{ij,1} x_{ij,1}}{l(e)} \qquad \forall e \in \mathcal{E}$$

where
$[\mathbf{w_e}]_\mathbf{c} \in \{\mathbf{0},\mathbf{1}\}$, $[\mathbf{y_n}]_\mathbf{c} \in \{\mathbf{0},\mathbf{1}\}$, $\mathcal{R}^{e}_{ij,1} \in \{0,1\}$, $x_{ij} \geq 0$, $0 \leq \psi \leq 1$

The above problem has $C(E+N)$ binary variables and $|\mathcal{T}|$ real variables. We now convert this non-linear mixed integer program into a linear mixed integer program, using some binary linearization techniques [45, 46, 47]. We note that, the following linearization techniques apply only to the fixed path routing case. For the single path routing scheme, the following techniques still lead to a non-linear formulation.

For each pair of logical links e and e', we define a $C \times C$ matrix $\mathcal{I}_{e,e'}$ as follows:

$$[\mathcal{I}_{e,e'}]_{ij} = [\mathbf{w_e}]_\mathbf{i}[\mathbf{w_{e'}}]_\mathbf{j}\mathbf{f_{e'}} \tag{3.29}$$

Using Equations (3.29) and (3.18), we have the following:

$$\left(\sum_{i=1}^{C}\sum_{j=1}^{C}[\mathbf{w_e}]_\mathbf{i}[\mathbf{LIM_{e,e'}}]_\mathbf{ij}[\mathbf{w_{e'}}]_\mathbf{j}\right) \times f_{e'} = \sum_{i=1}^{C}\sum_{j=1}^{C}[\mathcal{I}_{e,e'}]_{ij}[\mathbf{LIM_{e,e'}}]_\mathbf{ij} \tag{3.30}$$

The RHS of Equation (3.30) consists of $[\mathbf{LIM_{e,e'}}]_\mathbf{ij}$, which is a constant and $[\mathcal{I}_{e,e'}]_{ij}$, which is a real variable. We thus have a linear term on the RHS equivalent to the cubic term on the LHS. The only other non-linear expression, Equation (3.29) is equivalent to the following set of linear equations:

$$[\mathcal{I}_{e,e'}]_{ij} \leq [\mathbf{w_e}]_\mathbf{i} \tag{3.31}$$

$$[\mathcal{I}_{e,e'}]_{ij} \leq [\mathbf{w_{e'}}]_\mathbf{j} \tag{3.32}$$

$$[\mathbf{w_e}]_\mathbf{i} + [\mathbf{w_{e'}}]_\mathbf{j} - \mathbf{2} + \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}} \left(\mathcal{R}^{\mathbf{e'}}_{\mathbf{ij,1}} \times \mathbf{x_{ij,1}}\right) \leq [\mathcal{I}_{\mathbf{e,e'}}]_\mathbf{ij} \tag{3.33}$$

$$[\mathcal{I}_{e,e'}]_{ij} \leq 2 - [\mathbf{w_e}]_\mathbf{i} - [\mathbf{w_{e'}}]_\mathbf{j} + \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}} \left(\mathcal{R}^{\mathbf{e'}}_{\mathbf{ij,1}} \times \mathbf{x_{ij,1}}\right) \tag{3.34}$$

Using Equations (3.11), (3.12), (3.13), (3.19), (3.20), (3.18), (3.21), (3.28), (3.30), (3.31), (3.32), (3.33), and (3.34), we can replace the earlier non-linear formulation with its equivalent mixed integer linear program (Formulation 2).

This formulation has $C(E+N)$ binary variables, and $E^2C^2 + |\mathcal{T}|$ real variables. We note that the computation complexity of a mixed integer linear program, is only dependent on the number of integer variables [48]. The number of integer variables (binary variables in our case) in our formulation vary linearly with the number of nodes (N) and the number of edges (E). Hence, for small to medium networks, our formulation can be solved easily.

Given the optimal solutions for the channel assignment and flow allocation, we can assign the appropriate channels and interfaces to the mesh nodes.

3.5 Heuristic Algorithm for Channel Assignment and Flow Allocation

Given the network topology, our MILP formulation solves the joint channel assignment and flow allocation problem. The linear nature of the final formulation (Formulation 2), along with the less number of binary variables, reduces the computational complexity of the problem. However, in the worst case, this problem is NP-hard [14]. We thus propose a heuristic algorithm, that decouples the channel assignment from the flow allocation. We first propose a heuristic algorithm for channel assignment, and then perform flow allocation using the resultant channel assignment.

Formulation 2 Mixed integer linear program formulation for the joint channel assignment and flow allocation problem.

Maximize : $\psi \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}} \mathbf{x_{ij}} + (\mathbf{1}-\psi)\mathcal{Q}$

subject to

$$\sum_{i=1}^{C}[\mathbf{w_e}]_\mathbf{i} = \mathbf{1} \qquad \forall\, \mathbf{e} \in \mathcal{E}$$

$$\sum_{i=1}^{C}[\mathbf{y_n}]_\mathbf{i} \leq \mathcal{K} \qquad \forall\, \mathbf{n} \in \mathcal{N}$$

$$[\mathbf{y_n}]_\mathbf{c} \geq [\mathbf{w_e}]_\mathbf{c} \qquad \forall\, \mathbf{e} \in \mathcal{E}, \mathbf{n} \in \mathbf{Inc}(\mathbf{e}), \mathbf{1} \leq \mathbf{c} \leq \mathbf{C}$$

$$\sum \mathcal{R}^{e}_{ij,1} x_{ij,1} + \left(\sum_{e'}\left(\sum_{m=1}^{C}\sum_{n=1}^{C}[\mathcal{I}_{e,e'}]_{mn}[\mathbf{LIM_{e,e'}}]_\mathbf{mn}\right)\right) \leq c_0 \qquad \forall\, e, e' \in \mathcal{E}, e \neq e', (i,j,t_{ij}) \in \mathcal{T}$$

$$x_{ij} \geq \lambda t_{ij} \qquad \forall \text{ flows } (i,j,t_{ij}) \in \mathcal{T}$$

$$x_{ij} \leq t_{ij} \qquad \forall \text{ flows } (i,j,t_{ij}) \in \mathcal{T}$$

$$[\mathcal{I}_{e,e'}]_{ij} \leq [\mathbf{w_e}]_\mathbf{i}$$

$$[\mathcal{I}_{e,e'}]_{ij} \leq [\mathbf{w_{e'}}]_\mathbf{j}$$

$$[\mathbf{w_e}]_\mathbf{i} + [\mathbf{w_{e'}}]_\mathbf{j} - \mathbf{2} + \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}}\left(\mathcal{R}^{\mathbf{e'}}_{\mathbf{ij},\mathbf{1}} \times \mathbf{x_{ij,1}}\right) \leq [\mathcal{I}_{\mathbf{e},\mathbf{e'}}]_\mathbf{ij}$$

$$[\mathcal{I}_{e,e'}]_{ij} \leq 2 - [\mathbf{w_e}]_\mathbf{i} - [\mathbf{w_{e'}}]_\mathbf{j} + \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}}\left(\mathcal{R}^{\mathbf{e'}}_{\mathbf{ij},\mathbf{1}} \times \mathbf{x_{ij,1}}\right)$$

$$\mathcal{Q} \leq \frac{\sum_{(i,j,t_{ij})\in\mathcal{T}} \mathcal{R}^{e}_{ij,1} x_{ij,1}}{l(e)} \qquad \forall e \in \mathcal{E}$$

where

$[\mathbf{w_e}]_\mathbf{c} \in \{\mathbf{0},\mathbf{1}\}$, $[\mathbf{y_n}]_\mathbf{c} \in \{\mathbf{0},\mathbf{1}\}$, $\mathcal{R}^{e}_{ij,1} \in \{0,1\}$, $x_{ij} \geq 0$, $0 \leq \psi \leq 1$, $0 \leq [\mathcal{I}_{e,e'}]_{mn} \leq 1$

3.5.1 Channel Assignment

The channel assignment algorithm is presented in Algorithm 3. In this algorithm, $l(e)$ denotes the aggregate load on a link e, as defined in Section 3.4.3. $l_I(e)$ denotes the aggregate load in the interference region of a link e. We denote the set of links that interfere with a link e as $E_I(e)$, using which we have: $l_I(e) = l(e) + \sum_{e' \in E_I(e)} l(e')$.

Equivalently

$$l_I(e) = l(e) + \sum_{e' \in \mathcal{E}} l(e') \times ([\mathbf{w_e}]_\mathbf{c}[\mathbf{LIM_{e,e'}}]_\mathbf{cc}[\mathbf{w_{e'}}]_\mathbf{c}) \quad \forall\, e \in \mathcal{E} \tag{3.35}$$

In Equation (3.35), since we are not aware of the exact channels assigned to links e and e', we consider all links $e' \in \mathcal{E}$, that could potentially interfere with link e, if e and e' were assigned the same channel c, while calculating $E_I(e)$. The two parameters $l(e)$ and $l_I(e)$ represent two important characteristics of a link e in the network. $l(e)$ is the measure of the load on a link, while $l_I(e)$ captures the neighbourhood information. Links located in dense (with respect to node locations) regions of the network have a high value of $l_I(e)$ and vice versa. We use these two parameter values, available for each link, in our channel assignment algorithm. We define the interference weight $W_I(e,c)$ of a link e in channel c as:

$$W_I(e,c) = \sum_{e' \in \mathcal{E}} l(e') \times ([\mathbf{w_e}]_\mathbf{c}[\mathbf{LIM_{e,e'}}]_\mathbf{cj}[\mathbf{w_{e'}}]_\mathbf{j}) \tag{3.36}$$

where j is the channel assigned to link e'. In the algorithm, we handle the case when e' is not yet assigned a channel by initializing $[\mathbf{w_e}]_\mathbf{c} = \mathbf{0}\ \ \forall \mathbf{e} \in \mathcal{E}, \mathbf{1} \leq \mathbf{c} \leq \mathbf{C}$. A node n is said to be saturated if $\mathcal{K}$ number of distinct channels have been assigned to the links incident on node n, i.e., $\sum_{c=1}^{C}[\mathbf{y_n}]_\mathbf{c} = \mathcal{K}$.

Algorithm 3: Channel Assignment Heuristic Algorithm.

Input: A Graph $G(\mathcal{N}, \mathcal{E})$, with $l(e)$ and $l_I(e)$ values $\forall\, e \in \mathcal{E}$
Output: The channel assignment: $\bar{\mathbf{w}}_\mathbf{e} \quad \forall\, \mathbf{e} \in \mathcal{E}$
Initialize $[\mathbf{w_e}]_\mathbf{c} = \mathbf{0}\ \forall\, \mathbf{e} \in \mathcal{E}, \mathbf{1} \leq \mathbf{c} \leq \mathbf{C}$
While $\exists$ a link $e \in \mathcal{E}$ with $[\mathbf{w_e}]_\mathbf{c} = \mathbf{0}\ \ (\forall \mathbf{1} \leq \mathbf{c} \leq \mathbf{C})$ **do**
  If no node is saturated **then**
    Select the edge e' with the highest $l(e) \times l_I(e)$ value
    Select the channel c with the least $W_I(e', c)$ value
    Set $[\mathbf{w_{e'}}]_\mathbf{c} = \mathbf{1}$
    Update $[\mathbf{y_u}]_\mathbf{c} = \mathbf{1}$ and $[\mathbf{y_v}]_\mathbf{c} = \mathbf{1}$ where $e' = (u, v)$
    Check for node saturation of nodes u and v
  else
    while $\exists$ a node $n \in \mathcal{N}$, n is saturated **do**
     for all links e'', such that e'' is not assigned a channel and n is one of its incident nodes **do**
      Select c with the least $W_I(e'', c)$ value among the channels already allocated to links incident at node n
      Set $[\mathbf{w_{e''}}]_\mathbf{c} = \mathbf{1}$
      Check for node saturation of the other end node of link e''
     end for
    end while
  end if
end while

We note that heuristic for channel assignment has an order $O(E^2C)$. Thus, it is polynomial in nature.

3.5.2 Flow Allocation

After the first phase of the heuristic, we know the values of $\bar{\mathbf{w}}_\mathbf{e}$ and $\bar{\mathbf{y}}_\mathbf{n}$ $\quad\forall\,\mathbf{e} \in \mathcal{E}, \forall\,\mathbf{n} \in \mathcal{N}$. Our MILP formulation (Formulation 2) now reduces to an LP formulation which can be solved in polynomial time [49][50]. The simplified LP is shown in Formulation 3. In this formulation, the values of $\mathcal{C}_{e,e'}$ $\;\forall\, e, e' \in \mathcal{E}, e \neq e'$ are known in advance from the channel assignment. We note that our heuristic, consisting of the channel assignment algorithm and the flow allocation algorithm is polynomially bounded.

Formulation 3 Linear program formulation for the flow allocation problem.

Maximize : $\psi \sum_{(\mathbf{i},\mathbf{j},\mathbf{t_{ij}})\in\mathcal{T}} \mathbf{x_{ij}} + (\mathbf{1}-\psi)\mathcal{Q}$

subject to

$$\sum \mathcal{R}^{e}_{ij,1} x_{ij,1} + \left(\sum_{e'} \mathcal{C}_{e,e'} \times \sum \mathcal{R}^{e'}_{ij,1} x_{ij,1}\right) \leq c_0 \qquad \forall\, e, e' \in \mathcal{E}, e \neq e', (i,j,t_{ij}) \in \mathcal{T}$$

$$x_{ij} \geq \lambda t_{ij} \qquad \forall \text{ flows } (i,j,t_{ij}) \in \mathcal{T}$$

$$x_{ij} \leq t_{ij} \qquad \forall \text{ flows } (i,j,t_{ij}) \in \mathcal{T}$$

$$\mathcal{Q} \leq \frac{\sum_{(i,j,t_{ij})\in\mathcal{T}} \mathcal{R}^{e}_{ij,1} x_{ij,1}}{l(e)} \qquad \forall e \in \mathcal{E}$$

$$\sum_{m=1}^{C} \sum_{n=1}^{C} [\mathbf{w_e}]_\mathbf{m} [\mathbf{LIM_{e,e'}}]_\mathbf{mn} [\mathbf{w_{e'}}]_\mathbf{n} = \mathcal{C}_{\mathbf{e},\mathbf{e'}} \qquad \forall\, \mathbf{e}, \mathbf{e'} \in \mathcal{E}, \mathbf{e} \neq \mathbf{e'}$$

where

$\mathcal{R}^{e}_{ij,1} \in \{0,1\}$, $x_{ij} \geq 0$, $0 \leq \psi \leq 1$

3.6 Simulation Studies

In this section, we study the performance of our channel assignment and flow allocation algorithm. The parameters used in our simulation are specified in Table 3.1. In our simulations, we consider a wireless network with static nodes randomly located in a $1000m \times 1000m$ region. We choose a transmission range $TR = 200\,m$ and generate a topology from the random node placement, by adding a logical link between two nodes a and b, if they are within the transmission range of each other, i.e., $r_{ab} \leq TR$. We run our MILP on a network with $N = 20$ nodes. The maximum transmission power is set to $P^{Max} = 200\,mW$. We assume that all the nodes transmit

Table 3.1: Parameters used in the simulation.

Parameter	Value
Network Area	$1000\,m \times 1000\,m$
TR	$200\,m$
N	20
P^{Max}	$200\,mW$
N_0	-90 dBm
$SINR_{Min}$	$10\,dB$
G_{cd}	$\frac{1}{(r_{cd})^4}$
λ	0.1
c_0	$11\,Mbps$
C	11
$\mathcal{K}$	3

Table 3.2: Channels used in the simulation.

Number of Channels	Channels Used
1	6
2	$1, 11$
3	$1, 6, 11$
5	$1, 3, 6, 8, 11$
7	$1, 3, 4, 6, 8, 9, 11$
11	$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$

at P^{Max}. The thermal noise power is set to $N_0 = -90\,dBm$. The SINR threshold is set to $SINR_{Min} = 10\,dB$. The channel gain (G_{cd}) is set to $\frac{1}{(r_{cd})^4}$, where r_{cd} is the Euclidean distance between nodes c and d. The minimum guarantee for each flow is fixed at $\lambda = 0.1$. Each node is equipped with $\mathcal{K} = 3$ interfaces. The nominal data transmission rate c_0 is fixed at $11\,Mbps$. We consider a 802.11b network, and hence have a total of $C = 11$ channels, out of which 3 are non-overlapping channels, and the other 8 are partially overlapping channels. In our simulations, we generate flows with random source and destination nodes. We use the $GAMS/Cplex$ solver to solve our MILP.

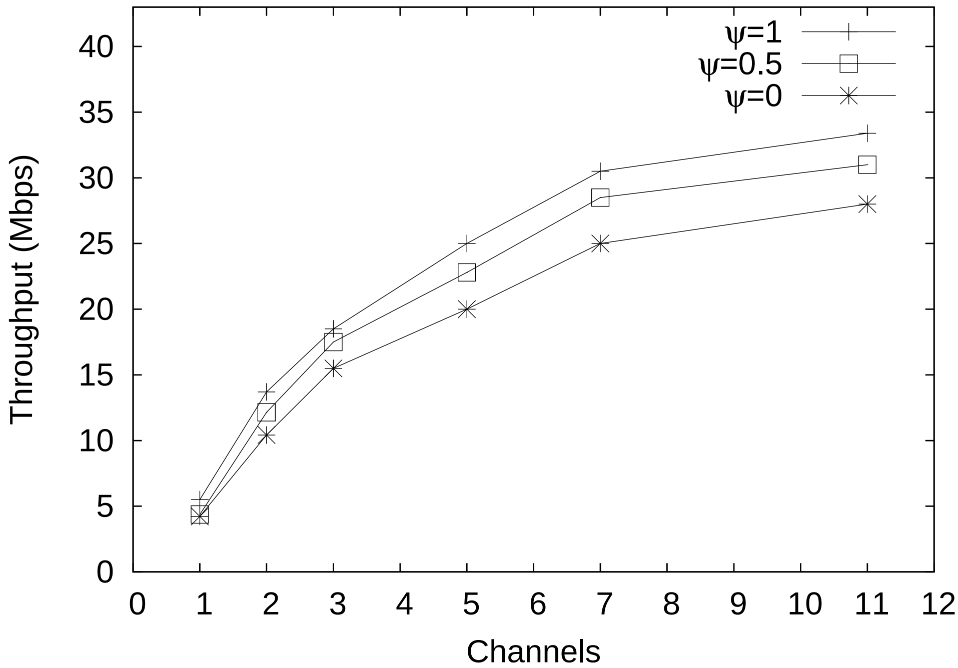

Figure 3.6: Variation of throughput with number of channels for a high load network.

3.6.1 MILP Performance

We now analyze the performance of our MILP formulation for the channel assignment and flow allocation problem. Figures 3.6 and 3.8, show the achieved aggregate throughput ($\sum_{(i,j,t_{ij})\in\mathcal{T}} x_{ij}$) for three different objective functions, with increasing number of channels, for a network with high and low load respectively. To simulate a network with a high load, the demanded throughput for flows is chosen between $[0.6c_0, 0.9c_0]$, and for a network with low load, the demanded throughput for flows is chosen between $[0.1c_0, 0.4c_0]$. Out of the three objective functions considered, the one with $\psi = 1$, corresponds to an objective that only considers aggregate throughput, the one with $\psi = 0.5$, gives equal importance to throughput as well as the queueing delay in the network, while the one with $\psi = 0$, only considers the queueing delay. Figures 3.7 and 3.9, show the average queueing delay per link $\left(\frac{1}{E} \times \sum_{e\in\mathcal{E}} \frac{f_{e,c}}{l(e)}\right)$, with the three different objective functions, with increasing number of channels, for a network with high and low loads respectively.

We note that, all the three objective functions considered, effectively utilize the additional channels available, by giving superior channel assignments. We see a steep increase in throughput with the addition of non-overlapped channels ($C = 1$ to $C = 3$). We also note that, even when partially overlapped channels are introduced ($C = 5$

to $C = 11$), our formulation still gives progressively better channel assignments that are able to attain higher aggregate throughput in the network. This is possible, because our formulation effectively utilizes the property of varying interference ranges to provide a channel assignment that makes efficient use of the spectrum available. We note that from $C = 5$ to $C = 11$, we are only introducing partially overlapped channels, without the use of any additional spectra (which would have been necessary if non-overlapping channels were to be added).

The objective that only considers throughput, performs best with respect to the aggregate throughput attained, at both high and low loads. However, from Figure 3.7 and 3.9, we can see that the same objective function, performs worst with respect to the average queueing delay. The objective that only considers throughput ($\psi = 1$), has the highest queueing delay, across all combinations of channels. Exact opposite behavior is seen in the objective function that only considers queueing delay. It performs the worst when aggregate throughput is considered, but performs best when average queueing delay is considered, for both high and low loads. We also note that with increasing loads, the difference between these two objective functions increases. Considering all these factors, in order to incorporate both our desired objectives, we use a weighted combination of the two objectives. The behavior of an objective function, that gives equal importance to both queueing delay and aggregate throughput ($\psi = 0.5$) is shown in Figures 3.6-3.9. When this objective function is used, our formulation tries to reduce the average queueing delay of the links, while trying to maximise the aggregate system throughput.

Figure 3.10, shows the variation of the aggregate system throughput with increasing number of radios, for $C = 5$ and $C = 11$. From the figure, we see that as the number of available radios in a node increases, the aggregate system throughput increases for both the cases. However, after a specific threshold, no increase is seen in the aggregate system throughput. This can be attributed to the degree of the nodes in the network. If the number of radios at a node is greater than its degree, then the additional radios become redundant, and they cannot be used for improving the system throughput (in our formulation, only one channel is assigned to each logical link).

The use of partially overlapped channels is an attractive replacement for additional non-overlapped channels. In Figure 3.11, we compare the aggregate system throughputs achieved, when 3 partially overlapped channels (channels $1, 3, 6$), 3 non-overlapped channels (channels $1, 6, 11$), and 11 partially overlapped channels (channels $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$), are used. In order to simulate medium loads, the

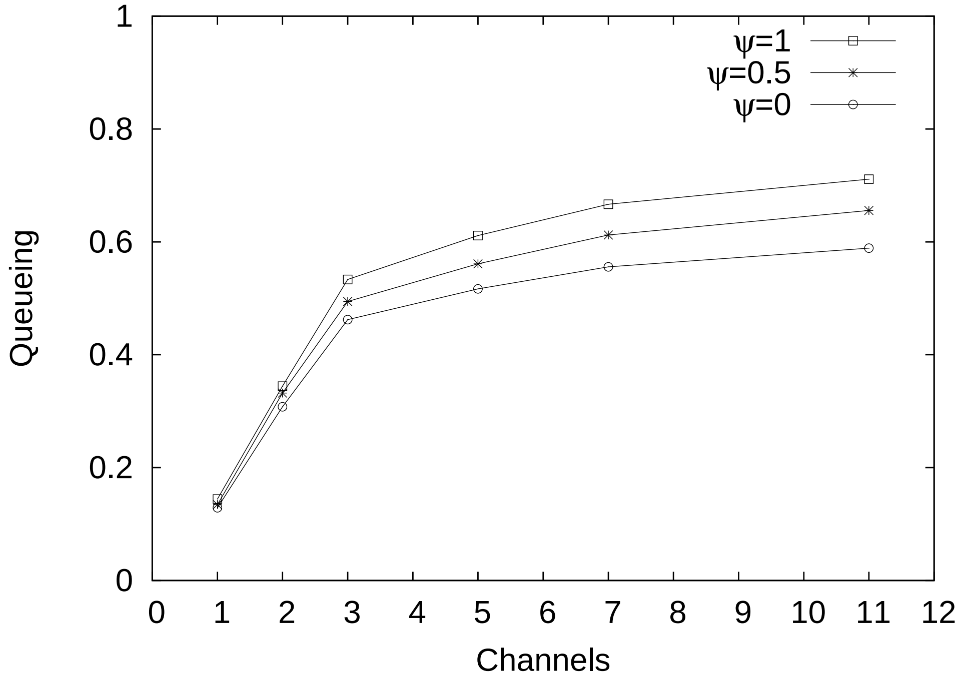

Figure 3.7: Variation of queueing with number of channels for a high load network.

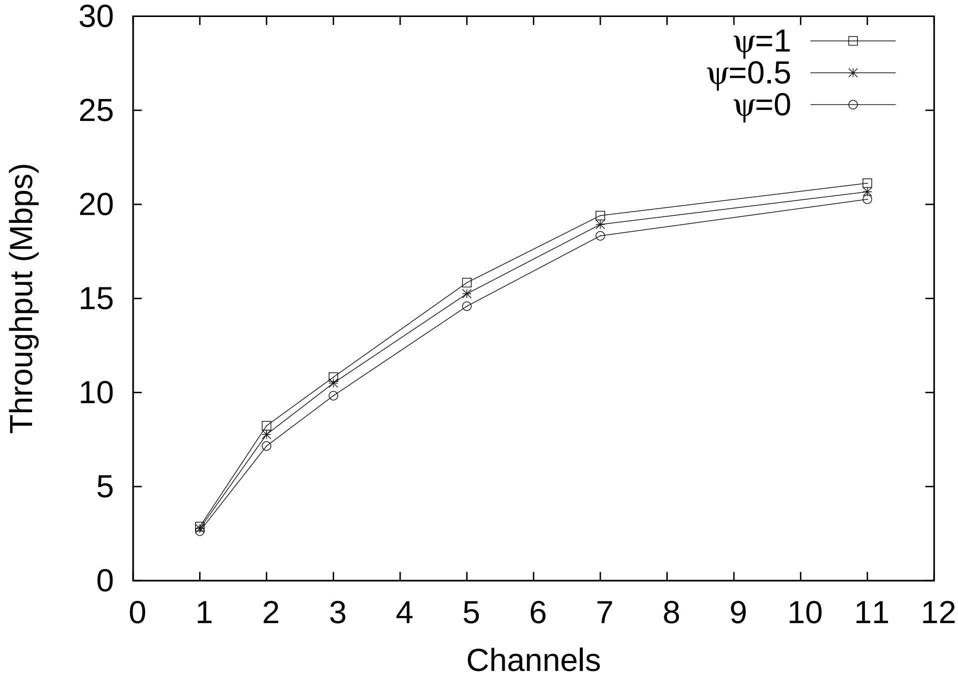

Figure 3.8: Variation of throughput with number of channels for a low load network.

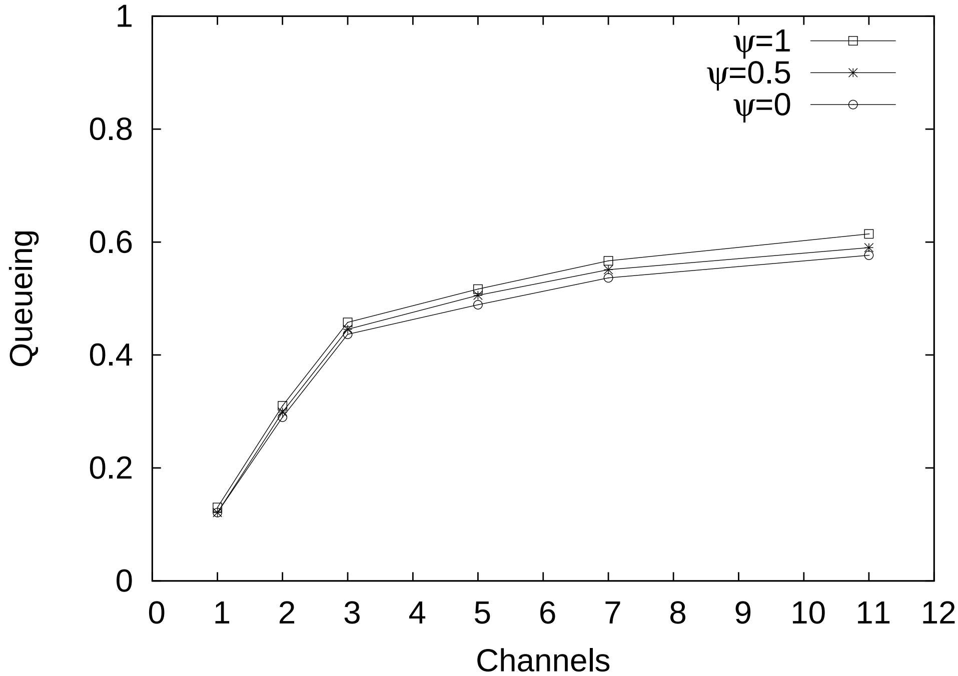

Figure 3.9: Variation of queueing with number of channels for a low load network.

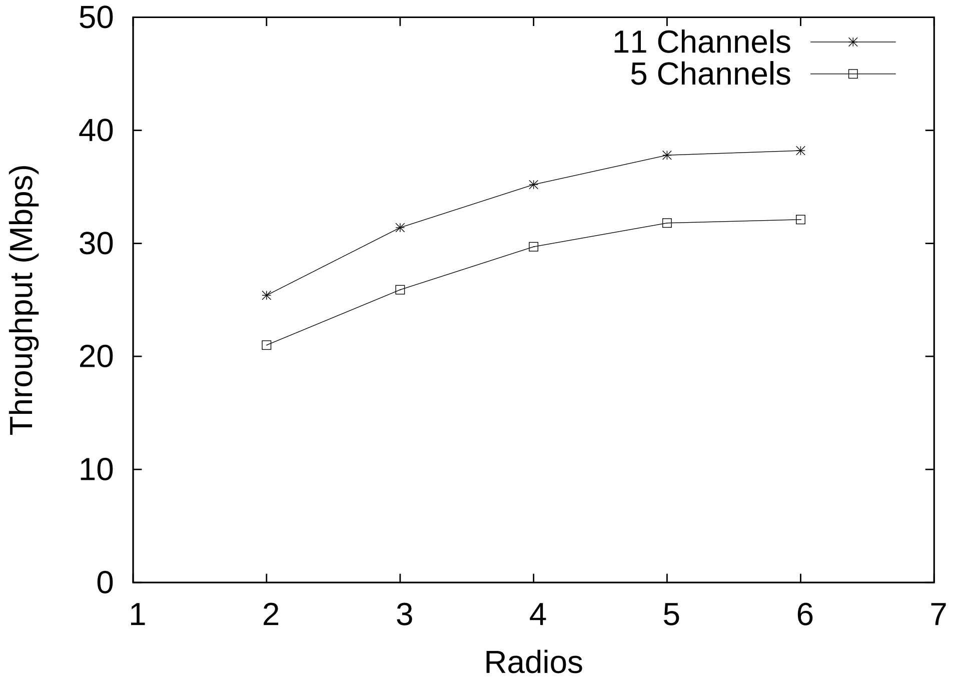

Figure 3.10: Variation of throughput with number of radios for a high load network.

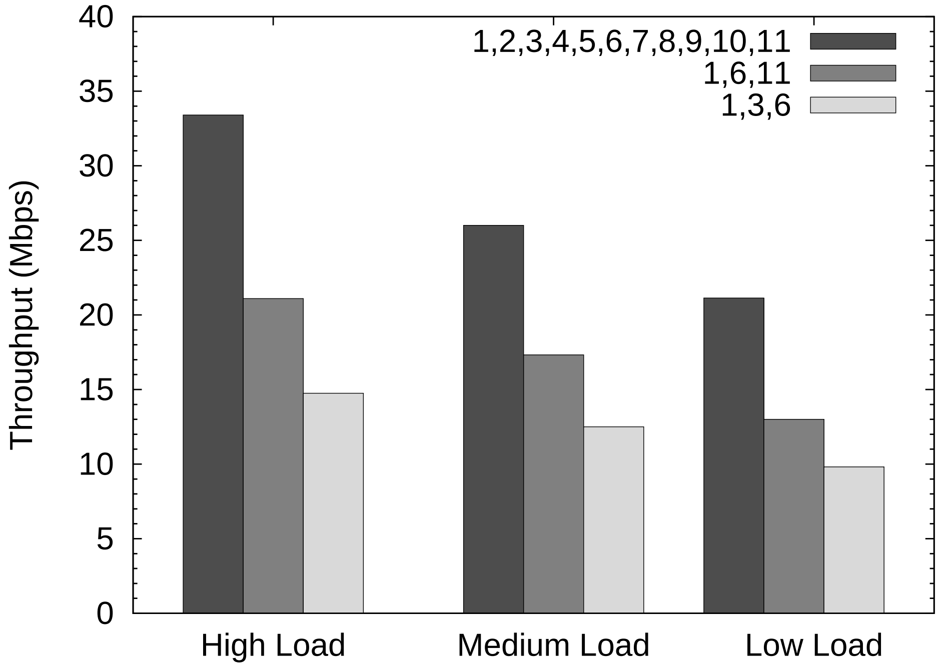

Figure 3.11: Comparing non-overlapped channels and partially overlapped channels.

demanded throughput for each communication session is uniformly chosen between $[0.1c_0, 0.9c_0]$. We see that, a system with 3 non-overlapped channels performs better than a system with 3 partially overlapped channels, as expected. However, we also see that with the addition of partially overlapped channels over the non-overlapped channels, without any additional use of the frequency spectrum, we are able to achieve a channel assignment with higher aggregate system throughput. This showcases the potential usefullness of partially overlapped channels. Every device which is compliant with the IEEE 802.11b standards, can be configured to be used on any one of the 11 partially overlapped channels in the frequency spectrum. Thus, without any additional support, systems can be configured to use our algorithm.

3.6.2 Heuristic Algorithm Performance

Figure 3.12 shows the performance of the heuristic algorithm with the MILP. The comparison is done, over a 20 node network, considering loads chosen randomly in $[0.1c_0, 0.9c_0]$. The values are obtained for 3 random network topologies, with the average throughput value being plotted in the figure. We note that, our heuristic performs on par with the optimal solution. The worst case performance of the heuristic, occurs for the case of $C = 11$, for which the percentage difference is only around

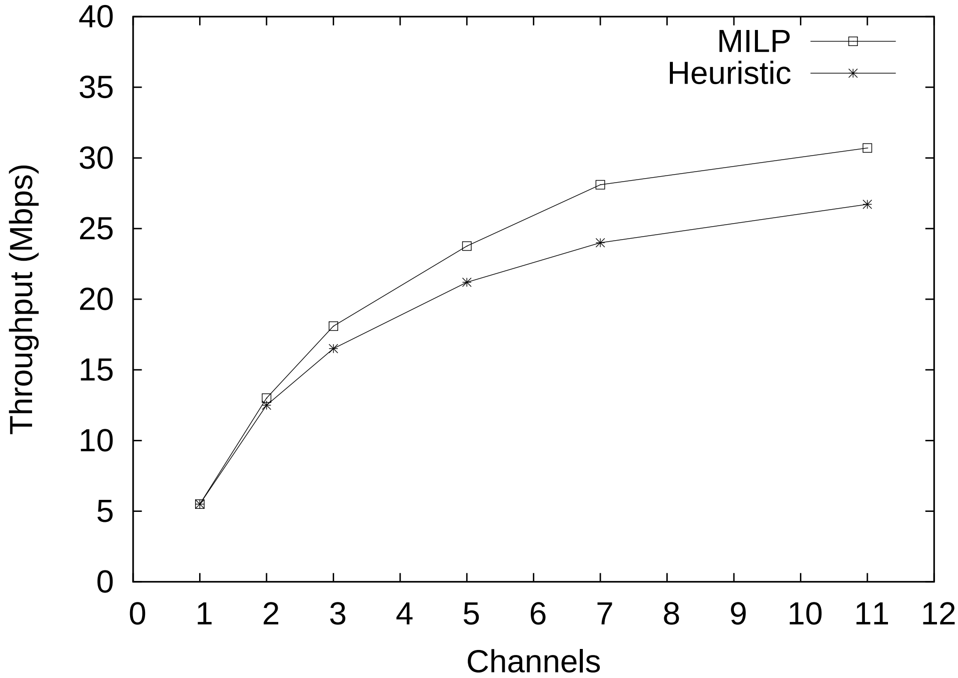

Figure 3.12: Comparison of the heuristic algorithm with the MILP.

13 percent.

We now study the performance of the heuristic over bigger networks. Figure 3.13 shows the rate of increase of aggregate throughput with network size, with increasing number of channels. We consider two random networks with $N = 15$ and $N = 30$ nodes, and plot the normalized aggregate throughput, where the normalization is done by dividing the aggregate throughput values with the throughput attained for the $C = 1$ case, for both network sizes. We assume random traffic chosen from $[0.1c_0, 0.9c_0]$ in both networks, and consider the average aggregate throughput attained over 3 different random traffic sequences. From the figure, we see that the rate of increase of the aggregate throughput with the addition of partially overlapped channels is higher for bigger networks. This stresses the importance of considering partially overlapped channels in bigger network topologies.

3.7 Summary

In this work, we propose a joint channel assignment and flow allocation algorithm for Multi-Channel Multi-Radio WMNs, when both non-overlapped and partially overlapped channels are being used. Our formulation takes into consideration several important network parameters such as the transmission power of each node, path loss

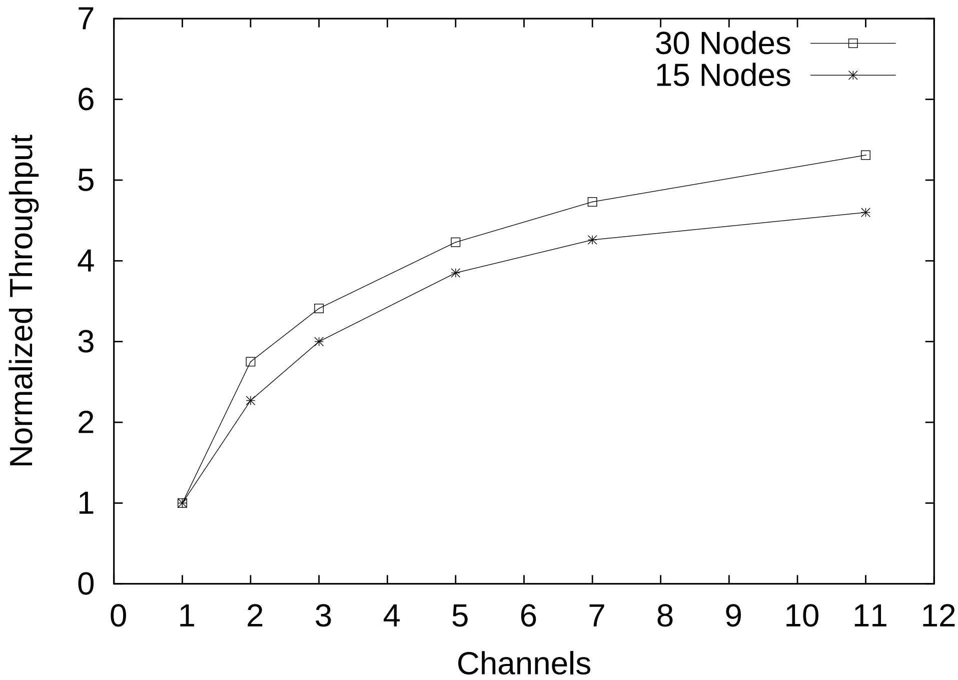

Figure 3.13: Rate of throughput increase with varying number of nodes.

information, the signal to interference plus noise ratio at a node, and the frequency response of the filters used in the transmitter and receiver. Since the traffic characteristics of a multihop WMN are quite different from a single hop wireless network, we consider maximizing aggregate end-to-end throughput and minimize queueing delay in the network, instead of the sum of link capacities. Through extensive simulations, we show that our formulation makes efficient use of the spectrum, by providing superior channel assignments and flow allocations, even with the addition of partially overlapped channels. In order to scale our algorithm for bigger network topologies, we also propose a polynomially bounded heuristic algorithm.

For future work, we plan to consider other routing schemes in our formulation. We also plan to extend our work by jointly considering, logical topology design, routing, and channel assignment in a unified framework.

CHAPTER 4

Conclusions and Future Work

4.1 Summary of Our Work

In this work, we have studied the achievable capacity of TDMA based WMNs and proposed efficient channel assignment and flow allocation algorithms for WMNs using both non-overlapped and partially overlapped channels.

We have theoretically analyzed the end-to-end call acceptance in TDMA based MC-MR WMNs. The estimates of the achievable capacity in terms of end-to-end call acceptance, allow us to answer questions such as the maximum number of high priority calls that can be accepted in the network, and the probability that the network enters into such a state such that no more calls can be accepted. We also considered providing bandwidth guarantees in the presence of resource contention and analyzed the network performance under different routing protocols with a distributed channel assignment scheme [16]. We considered a multi-dimensional Markov Process Model, and studied the effect of the number of radios in each node and the number of channels available in the network, on end-to-end probability of call acceptance (P_{Acc}). The effects of the routing protocol and the channel assignment algorithm were also incorporated into our theoretical bounds. From the theoretical bounds, we observe that if the channel assignment algorithm assigns equal number of edges for all channels, in each interference region, then the end-to-end call acceptance is maximum. The routing protocol that uses the WCETT metric (MR-LQSR) out performs SP in terms of the probability of call acceptance. This clearly indicates the importance of considering path based interference in the routing protocol.

We then looked into the issue of improving the system throughput of WMNs, when both non-overlapped channels and partially overlapped channels are used. We proposed a joint channel assignment and flow allocation algorithm for MC-MR WMNs, which takes into consideration several important network parameters such as the transmission power of each node, path loss information, the signal to interference plus noise ratio at a node, and the frequency response of the filters used in the transmitter and receiver. Since the traffic characteristics of a multihop WMN are quite different from a single hop wireless network, we consider maximizing aggregate end-to-end throughput and minimize queueing delay in the network, instead of the sum of link capacities. Through extensive simulations, we show that our formulation makes efficient use of the spectrum, by providing superior channel assignments and flow allocations, even with the addition of partially overlapped channels. In order to scale our algorithm for bigger network topologies, we also propose a polynomially bounded heuristic algorithm.

4.2 Future Work

4.2.1 Achievable Capacity of TDMA based WMNs

- Achievable Capacity measures the number of calls that are accepted by the network. However, an accepted call may not go to completion. Changes in network configuration due to interference, may cause call dropping. Effects of such changes have not been considered in our analysis.
- In order to provide deterministic guarantees for high priority calls, preemption of lower priority calls is necessary. If preemption is allowed, the call acceptance for the lower priority calls decreases. Allowing preemption of calls raises several issues, such as the behavior of call acceptance probability and the dependence on the preemption scheme.
- In our analysis, the transmission range of a node is modeled as a circle, centered on the node and with a radius equal to the node's transmission range. The analysis could be improved if this assumption is relaxed.
- The case of a single class of calls has been considered in this work. The next step is to consider the effect of multiple classes of calls. The simulation studies must also be extended by considering other protocols, in order to infer the essential properties of protocols that approach optimal behavior. Further work can be done to improve the theoretical bounds.

4.2.2 Channel Assignment and Flow Allocation for WMNs with Partially Overlapped Channels

- In our formulation, we have considered the fixed path routing scheme. The formulation can be extended to other types of routing schemes specified in our work.

- Our work can be extended by jointly considering, logical topology design, routing, and channel assignment in a unified framework.

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